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A SYSTEMS ENGINEERING APPROACH TO MULTIPLE ATTRIBUTE UTILITY TH--ETC(U)
JAN 80 A R DEWISPELARE
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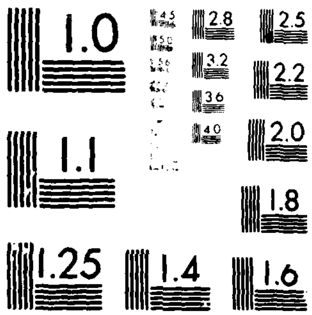

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ABSTRACT

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Process algorithms for multiple objective optimization theory (MOOT) and multiple attribute utility theory (MAUT) are presented at a theoretical level as well as at the application level in a systems engineering framework appropriate for choice making. A subsequent comparison of the two approaches motivated a combined MOOT/MAUT methodology which utilizes, in an efficient manner, the complementary aspects of both processes.

Results are presented of the application of this joint approach to a defense systems acquisition problem. Specifically, a paradigm for electronic warfare aircraft retrofit was developed using the combined multicriteria MOOT/MAUT process. A set of criteria was developed in this application which can be used to evaluate, in a comprehensive manner, alternative system configurations.

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A SYSTEMS ENGINEERING APPROACH TO MULTIPLE
ATTRIBUTE UTILITY THEORY AND MULTIPLE
OBJECTIVE OPTIMIZATION THEORY: WITH APPLICATION TO
AIRCRAFT RETROFIT DESIGN

A Dissertation
Presented to
the Faculty of the School of Engineering and Applied Science
University of Virginia

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy (Systems Engineering)

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by

Capt. Aaron R. DeWispelare, USAF

January 1980

APPROVAL SHEET

This dissertation is submitted in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy (Systems Engineering)

Author

This dissertation has been read and approved by the Examining Committee:

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January 1980

The opinions expressed here are those of the author and do not necessarily reflect the views of the Department of Defense, any specific military service, or any individual within the Defense establishment.

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EXECUTIVE SUMMARY

Introduction

The ever increasing complexity of decision situations coupled with the requirement to cope with the political, economic, social and technical aspects of these situations has resulted in considerable interest being given to the multiple criteria decision theory (MCDT) approaches of multiple objective optimization theory (MOOT) and multiple attribute utility theory (MAUT).

MAUT is a branch of decision theory which is designed to result in decision making aids for situations incorporating multiple criteria. MOOT incorporates a set of techniques which were developed to be computationally efficient optimization tools for problems involving multiple criteria. Since MOOT and MAUT were developed for different purposes, it is not unexpected that each approach is proficient at different stages in the resolution of a decision situation. This difference in proficiency motivates the development of a combined MOOT/MAUT approach which realizes increased efficiency through the utilization, in a complementary manner, of appropriate parts of MOOT and MAUT.

An appropriate application for the multiple criteria approach developed in this research is a specific military equipment acquisition involving aircraft retrofit. The retrofit of a particular aircraft with equipment designed for a mission which the aircraft was not originally designed to fly typically requires a large systems effort. Specifically, the retrofit of an aircraft with sophisticated electronic warfare (EW) equipment has historically involved inefficiencies and inadequacies including schedule and budgetary overruns and a lack of initially specified final product performance. Development of a useful combined MOOT/MAUT process seems a logical choice to ameliorate the difficulties of current electronic warfare aircraft retrofit design (EWARD) processes which are not unique in defense equipment acquisition.

The major contributions of this effort are the delineation of systems engineering process algorithms of MOOT and MAUT (pp 23-32,49-56); the development of a combination MOOT/MAUT methodology based on the

complementary characteristics of both approaches (pp 69-77); the development of an efficient framework for EWARD through extension of a MCDT approach to that application (pp 98-143); and the generation of a set of criteria for evaluation of alternative retrofit systems in the defense systems acquisition cycle (pp 98-113).

Approach

The dissertation is divided into two interrelated efforts. The first part is concerned with examining two approaches to multiple criteria decision theory with a goal to investigate where processes resulting from application of the theory could be made more efficient. First, MOOT and MAUT are presented at a theoretical level, and then the process algorithms for the two approaches are delineated with an analyst/user orientation. A subsequent comparison of the two approaches with respect to structure, purpose, and function, shows that the MOOT approach is particularly efficient at the optimization step in the resolution of a decision situation, while MAUT is proficient at the ranking of alternatives/decision making step. The MOOT process was found to often need easily usable techniques for amalgomating the objective functions into a scalar choice function and incorporating the preference structure of the DM. The MAUT process was found to be potentially troublesome to DMs because of the extensive elicitation process required to form the scalar scoring function. Many of the attributes of the MAUT process are needs of the MOOT process. The converse case of the attributes of MOOT and needs of MAUT is also true. So the results of the above mentioned comparison motivated the development of a joint MOOT/MAUT approach. The DELTA chart of Figure 1 shows the algorithm for this joint MOOT/MAUT approach. The primary benefit of the combined MOOT/MAUT process over the application of either MOOT or MAUT individually is that an increase in efficiency is realized as this joint approach utilizes the best of MOOT in the optimization step and the best of MAUT in the decision making step. This efficiency is realized in a complex decision situation as a savings in time and resources through the systematic application of the part of processes specifically for the purposes they are best suited.

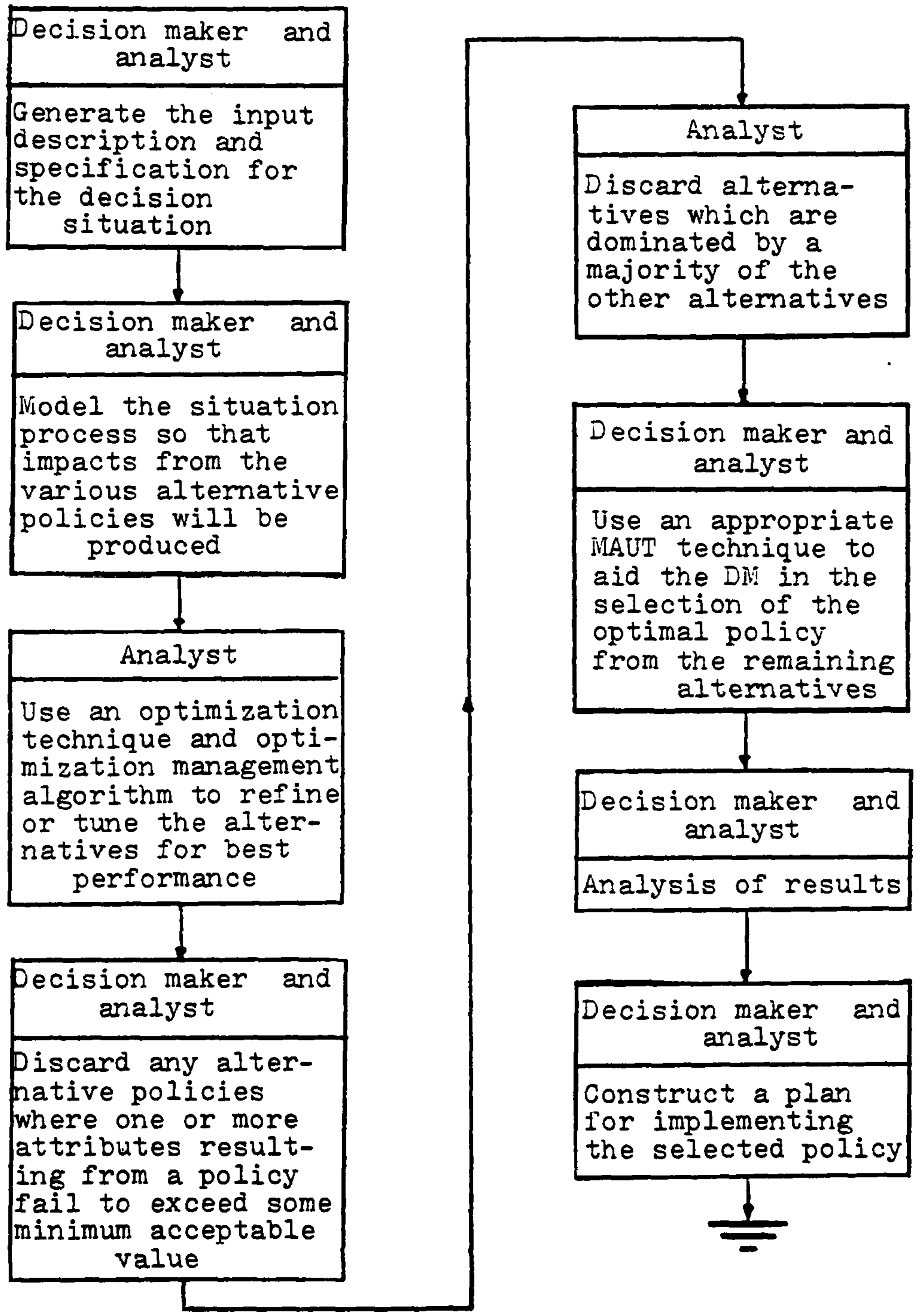

Figure 1. Abbreviated DELTA Chart Of The Combined MOOT/MAUT Algorithm

The second major accomplishment of the research is in deducing whether or not a MCDT approach could ameliorate inefficiencies extant in a particular defense systems equipment acquisition. The specific application entailed applying the combined MOOT/MAUT approach to an EWARD situation in the Conceptual Phase of the Defense Systems Acquisition Cycle. A pre-analysis phase was conducted in which three primary groups of stakeholders were identified. These groups were the operations group, the government policy group, and the technical development and assessment group within the U.S. Air Force.

A set of twenty-one decision makers (DM's) and advisors from the three groups identified above volunteered to take part in this effort. These participants were individuals who were currently involved in the design, production and procurement of EW equipment. Interaction with the participants was through a series of interviews. As expected, the objectives identified by the participants were found to be non-commensurable in the sense that no common measure (cost, volume, etc.) could be found. With the interaction of the DMs and advisors from the three stakeholder groups, a set of criteria which included the salient attributes of EWARD was established (Table 1). A set of alternative systems indicating the impacts of retrofitting the alternative systems with respect to levels of the attributes was obtained from government and industrial sources. Data used in our example has been modified to incorporate the realism of a retrofit situation without identifying specific equipment. Preferences, utilities, and minimum acceptable attainment levels of the attributes were elicited from the groups to be used in the MOOT/MAUT process. Through utilization of the MOOT/MAUT process, the decision making group was able to identify an optimal alternative in an expedient manner. A validation exercise was performed on an actual system now in use to corraborate the developed EWARD approach as an efficient process for identifying a satisfactory retrofit configuration.

Two important features seen in this application effort are that this application methodology makes provisions for participation of decision makers, advisors, and experts at an early phase of EWARD, and the methodology used is flexible enough to be applicable to any EWARD

Table 1

Criteria for an EW Retrofit System

1. Technical: EW Aircraft Aerodynamic Performance
   - a. EW System Weight
   - b. EW System Volume Required
   - c. EW System Power Required
2. Economic: EW Retrofit System Life Cycle Cost
3. Military: Retrofit System Electronic Warfare Performance
   - a. Aircrew Performance
   - b. Number of Threats Degraded
   - c. Number of Threats Defeated
4. Political: National Policy Satisfaction

of the near future. An EWARD can be accomplished by use of the attribute template determined in this research together with the MOOT/MAUT algorithm which was developed.

Conclusions

Based on this research, the following conclusions can be drawn:

- While there are operational and philosophical differences between MOOT and MAUT, both processes are mental constructs to approaching decision situations, and when they are compared at the same level, there are for all intents and purposes, no essential differences in structure between them. At the application level, both MOOT and MAUT approaches will allow identification of a strategically equivalent optimal policy, assuming the DM is consistent and the NDSS is complete.
- The complementary phases of MOOT and MAUT are compatible for combination into a single methodology.
- A MCDT approach has merit in an EWARD application, particularly in the early stages of this application. The combined MOOT/MAUT approach should increase the efficiency of EWARD in a comprehensive manner for an overall time and resource savings. Table 2 lists the impacts of this EWARD application as determined in this research. An optimal system configuration, (no. 8, Table 5, Chapter 5) was identified in a timely manner.
- The multiplicative form of scoring function for evaluation of alternative configurations is an appropriate form in EWARD to insure sensitivity of the scoring form to small differences in the alternatives.
- The experimental subject group used in this research viewed the MOOT/MAUT framework presented as an acceptable and desirable approach to this specific defense systems equipment acquisition situation (EWARD).
- Careful assessment of preferences and cooroboration of the scaling constants in the aggregated utility functions of DMs and advisors is critical to identify the optimal system configuration correctly.

Table 2

Comments Concerning Application of the MOOT/MAUT Process to EWARD

Group	Comments
Operations Group	A final product is procured which should perform in a satisfactory manner
	An interchange of ideas concerning the retrofit system occurs early in the acquisition process
	A needed capability is added to the operational inventory at an early date
Government Policy Group	The Development Concept Paper (used for system evaluation) is presented in a more meaningful format (utilizing the developed criteria)
	Management is required of the analysis team conducting the EWARD
	Other R & D programs should benefit from the time and resources saved
	An interchange of ideas concerning the retrofit system occurs early in the acquisition process
Technical Development and Assessment Group	An interchange of ideas concerning the retrofit system occurs early in the acquisition process
	A savings of time and resources required in the acquisition process is realized (up to one year of development time in the Conceptual Phase alone can be saved compared to the present procedure)
	A common set of criteria on which to base testing and evaluation is now available

Recommendations

Based on this research and application effort, the following recommendations are offered:

- A combined MOOT/MAUT methodology should be implemented in EWARD in the Conceptual Phase of the Defense Systems Equipment Acquisition Cycle. This process should be implemented utilizing the available analytic and technical staffs of the identified stakeholder groups. For convenience, it is suggested that these staffs be organized in a single team mode under the direction of the Office of the Secretary of Defense.
- The set of developed criteria should be utilized in EWARD as a comprehensive means to evaluate retrofit system configurations.

As a result of our efforts, we can identify the following areas of further research. There is a need to further develop techniques for selection of the optimal policy from a non-dominated solution set in ways that facilitate decision maker-analyst interaction. On the whole, this important area has received little effort from the MOOT advocates. Another area for further investigation concerns the need for more adequate modelling of the behavioral aspects of decision situations so that by incorporating a DM's cognitive style, there would result increased DM acceptability of the analytic approach and the results. Additional research aimed at examining mathematically the preference space at pre-optimization (for MAUT) and post-optimization (for MOOT) would aid in the steps of modelling, ranking alternatives, and decision making for multiple criteria decision situations. Efforts are also needed to further characterize the stated objective of national policy satisfaction as it relates to EWARD.

ABBREVIATIONS AND ACRONYMS

AFLC	Air Force Logistics Command
AFR	Air Force Regulation
AFSC	Air Force Systems Command
ASD	Aeronautical Systems Division (AFSC)
DCP	Development Concept Paper
DM	Decision Maker
DOD	Department of Defense
DODD	Department of Defense Directive
DSARC	Defense Systems Acquisition Review Council
DT & E	Development Test and Evaluation
ECP	Engineering Change Proposal
ECR	Extended Contributive Rule Method
EW	Electronic Warfare
EWARD	Electronic Warfare Aircraft Retrofit Design
FDSD	First Degree Stochastic Dominance
G-1	Group 1, (Operations,Intelligence)
G-2	Group 2, (Government Policy)
G-3	Group 3, (Technical Development and Assessment)
GOR	General Operational Requirement
HQ. USAF	Headquarters USAF
IOT & E	Initial Operational Test and Evaluation
ISM	Interpretive Structural Modelling
MANACON	Computer Programs for Decision Analysis
MAUT	Multiple Attribute Utility Theory
MCDM	Multiple Criteria Decision Making
MCDT	Multiple Criteria Decision Theory
MADA	Multiattribute Decision Analysis
MDI	Mutual Difference Independence
MENS	Mission Element Need Statement
MI/MCDM	Multiple Independent Entity/MCDM
MOOT	Multiple Objective Optimization Theory
MSP	Multiobjective Stochastic Programming
MUI	Mutual Utility Independence
MWDI	Mutual Weak Difference Independence
NATO	North Atlantic Treaty Organization
OMB	Office of Management and Budget
OMBC	Office of Management and Budget Circular
OSD	Office of the Secretary of Defense
PCA	Physical Configuration Audit
PI	Preferential Independence
PMD	Program Management Directive
PMP	Program Management Plan
R & D	Research and Development
RECON	Reconnaissance
RFD	Request for Proposal
SAC	Strategic Air Command
SAL	Strategic Arms Limitation Talks
SDSD	Second Degree Stochastic Dominance

SM	Systems Management Team
SON	Statement of Need
SP	Stochastic Programming
SPO	System Program Office
(S)SARC	(Service) Systems Acquisition Review Council
TAC	Tactical Air Command
UI	Utility Independence
USAF	United States Air Force
WDI	Weak Difference Independence

GLOSSARY

Activity - Specific utilization of resources in accordance with a policy or decision

Advisor - An individual who is utilized in an advisory capacity by a decision maker

Alternative - A candidate course of action
(Alternative Solution)
(Alternative Action)

Attribute - The performance measures of objective attainment measured on outcomes

Consistent Decision Maker - The preference structure of the decision maker is identical throughout the resolution of the decision situation

Constraints - The restrictions generally concerning resources which cause the necessity of optimizing

Criterion - A standard, rule, or test by which a judgment can be formed on the goodness of an alternative solution; an objective function or cost function can be used as a criterion

Decision - An allocation of resources by the decision maker as a solution to the decision situation; it is an expression of preference for a particular alternative from a class of alternatives

Decision Maker - An individual who has the authority to make a policy decision, and implement activities concerning that decision

Defeat (a threat) - Render a threat system inoperable

Degrade (a threat) - to reduce effectiveness of the threat system by some degree

Element - A constituent part of the decision situation (i.e., objective, alternative, decision maker, etc.)

Event - An occurrence that is characterized as a state of nature

Non-dominated (non-inferior) solution space - The form of the efficient solution space which results from optimization of a vector criteria of a realistic problem with conflicting objectives. An allocation of resources is non-dominated if no other reallocation of resources will increase the value of any of the constituent criterion without decreasing the value of at least one component criterion

Objective - An incentive which reflects the value system of the decision maker and upon which the decision situation is based; an objective is expressible as the "infinitive to plus a verb plus an object"

Objective function - A criterion in functional form made up of attri-
(Cost function) butes which is used to evaluate candidate solutions

Objective (or Goal) - The attributes of a decision situation
Measures

Outcome - A consequence of a decision which is a combination of alternatives and events; the outcomes characterize the "state" in certain decision situation formulations

Pareto optimality - A solution is Pareto optimal when the solution is adjusted to the point such that no change can be made to increase the utility of any individual without decreasing the utility of others

Pre-analysis phase - The initial efforts in decision situation resolution composed of the steps of problem definition, value system design, and system synthesis

Preference - A ranking dependent on one's opinion or choice

Risk - The stochastic situation where the probability density function of the states of nature is known

Structure of a decision - The identification and organization of com-
situation ponents or elements of the decision into a framework or situation model for determining the optimal course of action

Threat - An opposing weapon system

Quantifiable impact - An impact which has a direct numerical measurement (i.e., volume, temperature, etc.)

Utility - The usefulness of something or satisfaction derived from something; an individual expresses cardinal utility when he or she assigns to an item a number representing the amount or degree of utility associated with it. Ordinal utility requires an individual to assign numbers to items which only provide a ranking or ordering of preferences for those items

Value - the worth or desirability of a thing as evaluated by the decision maker

Value Function (Cardinal) - A mathematical function which relates the worth of something to the decision maker in a way which is measurable and order preserving up to a positive linear transformation

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CHAPTER 1

A PERSPECTIVE ON MULTIPLE CRITERIA DECISION MAKING

1. Introduction

Policy and decision makers, at all levels of authority in all segments of the public and private sector, are confronted with a variety of decision situations. Decision and policy analysts are constantly seeking better techniques to aid the DM to resolve these decision situations in an efficient manner. Complex decision situations should generally be approached using a multiple criteria format since there are often issues from the technical, social, economic, legal, political, military and other sectors which need to be considered. Analysts who aid DMs have often approached multiple criteria decision situations using either of two methods: multiple objective optimization theory (MOOT) and multiple attribute utility theory (MAUT). These two approaches are discussed by Cochrane and Zeleny (1973) and MacCrimmon (1973).

These two approaches require a structuring of the decision situation. This structuring process generally consists of identifying and organizing components and factors of the decision situation into a framework or situational model for determining an optimal course of action (Sage and Rajala, 1978). The systems engineering methodology of Hall (1969) has provided a useful framework for the structuring, design, and analysis of large-scale systems. Certain of the steps in Hall's framework can be accomplished utilizing either MOOT or MAUT to produce an optimal solution. This leads us to investigate the processes involved when utilizing either approach (MOOT or MAUT) in order to compare them critically with respect to structure, function, purpose, and organizational implication. Our investigation of MOOT and MAUT is motivated by a desire to combine the best of both processes for a more efficient multiple criteria decision making approach. This resulting combined MOOT/MAUT approach, especially after fuller development to include detailed treatments of risk, stochastic dominance, and other affects, should have application in a number of large-scale decision situations.

This combined MOOT/MAUT approach will be applied to an electronic warfare aircraft retrofit design (EWARD) problem. This application effort will be aimed at producing planning and design prospectives for EWARD type issues. The retrofitting of special purpose military aircraft is a large-scale problem because of the political, economic, and technical overtones. Previous retrofitting approaches considering primarily the technical aspects of the problem have only met with limited success (Peterson, Hays, and O'Conner, 1975).

In this chapter, a discussion of the characteristics of any general decision situation is presented following a discussion of related efforts. Next, a section on multicriteria approaches, including an introduction of the two approaches of MOOT and MAUT, is presented followed by a discussion of the systems engineering methodology. The purpose of this research is not to present theoretical or pragmatic discussions concerning development of a DM's value system or elicitation of an associated preference and utility structure as there are examples of these in the literature. Our effort is aimed at incorporating the results of a value system and preference/utility elicitation into a MCDT structure for the development of an efficient decision making framework, and this part of the effort is emphasized.

Related Efforts

Previous efforts concerned with the resolution of decision situations involving multiple criteria have generally concentrated on one or the other of MOOT and MAUT as evidenced by the literature. Cohon and Marks (1975) divide the popular optimization techniques into classes which can be used for formulation of multiobjective solutions. Slovic, Fischhoff, and Lichtenstein (1977) provide insight into the many facets of both normative and descriptive behavioral decision theory. Huber and Johnson (1977) and Edwards (1977) provide outlines for modelling and optimizing MAUT situations. The decision making process in terms of MCDM is discussed by Starr, et al.,(1977). Brown and Ulvila (1976) segment decision problems under three taxonomic headings of decision situation, analysis, and performance measure. Cochrane, et al. (1973),

discuss several variations of both MOOT and MAUT techniques. The effect of hierarchial decision making on the organization is discussed by Banker and Gupta (1978). Merkhofer (1977) discusses the concept, advantages, and value of flexibility in a decision situation. The works of Keeney and Raiffa (1976), and Raiffa (1968) are decision analysis classics concerning MAUT.

The unclassified literature on the EWARD application is well represented in the work of Peterson, Hays, and O'Connor (1975) and Cook (1977). This effort is aimed at suggested modelling formulations for a multiple criteria decision theory (MCDT) approach to electronic warfare equipment selection.

The systems engineering concepts of Hall (1969), Hill and Warfield (1972), Gibson (1977), and Sage (1977) influenced the philosophical approach to the decision situation research and EWARD application effort.

2. The Decision Situation

A decision situation contains a set of elements which are dependent on the circumstances of the problem facing a decision maker (DM). These elements are utilized in the pursuit or selection of a decision alternative as a solution to a problem. The decision situation is composed of various components surrounding the problem such as the internal organization, external (physical) environment, and rationale for determining decisions. The selection and implementation of an appropriate (optimal) policy is the desired resolution of a decision situation. As one of the first steps in a decision aiding effort, the analyst attempts to structure and model the decision situation incorporating the DM's value system. Figure 1 shows some elements of a typical decision situation and the steps of the systemic process which will be used in the evolution of a choice making strategy.

3. *Multicriteria Decision Making*

The term "multiple criteria decision" is applied to a decision problem involving either multiple objectives or multiple attributes. There is a need for MCDT in numerous fields and applications areas (Keefer, 1978; Edwards, 1977; and Slovic, et al., 1977). A vector of

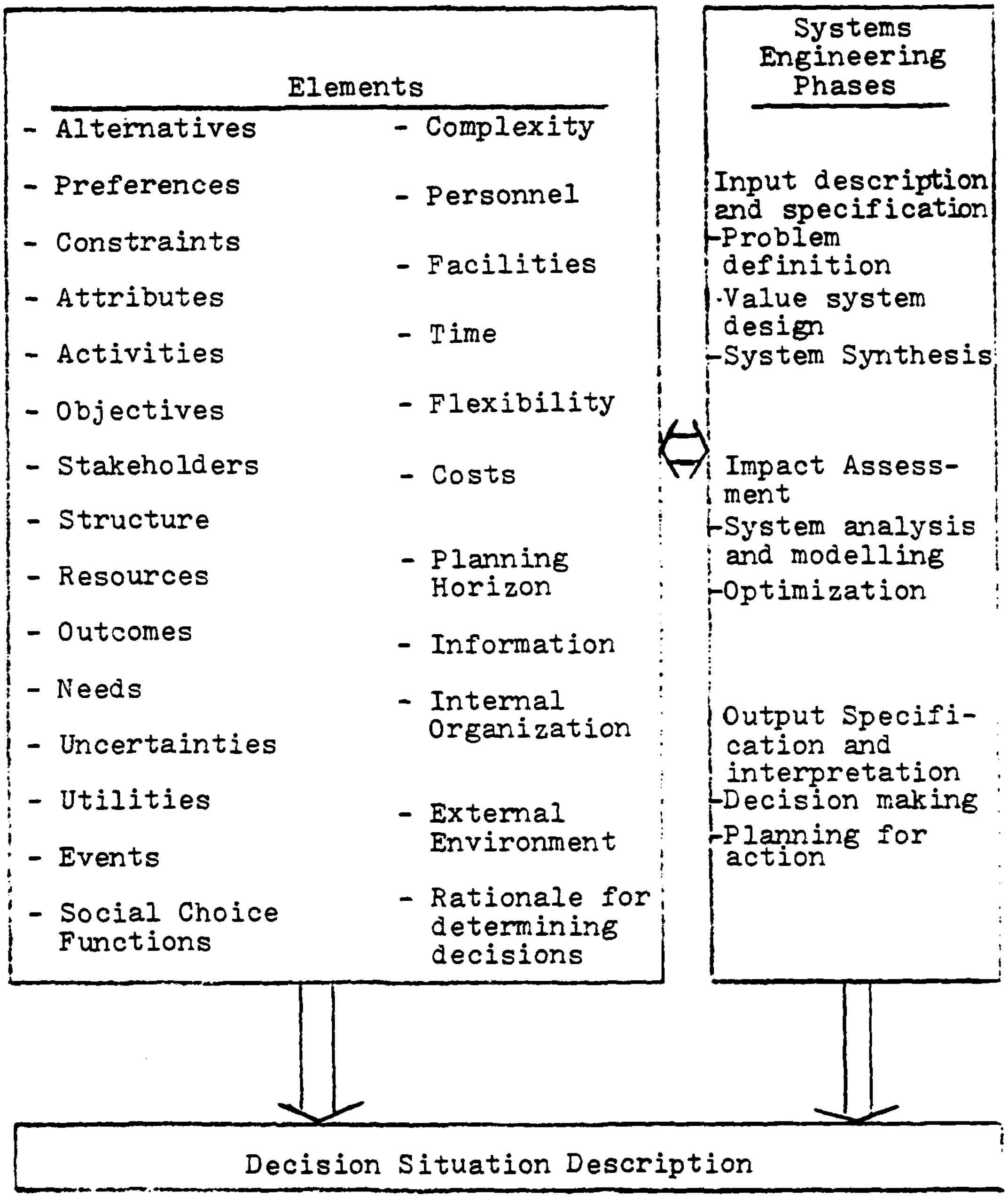

Figure 1. Decision Situation Components

objectives, for either MOOT or MAUT is needed because of the need to consider multiple non-pecuniary and non-commensurate objectives (Haimes, Hall, and Freedman, 1975). Even when fundamentally independent objectives are combined in a decision situation with limited resources, the objectives become dependent and often conflicting with respect to purpose. Single criterion models and techniques, which often consider economic objectives only, are not adequate in contemporary government, business, industrial, and societal decision situations which require inclusion of non-pecuniary objectives (Hoos, 1972; Sutherland, 1978; Haimes, Hall and Freedman, 1975). The political, economic, social, and technical ramifications of large-scale system decisions require inclusion of pertinent attributes and the value system of the stakeholders for truly satisfactory problem resolution. Systems Engineers attempt to fill the void between idealistic mathematically tractable decision theoretic problems and the complex multidiscipline real world situation of the modern day decision maker in order to evolve a set of tools and techniques which can provide contextural solutions to decision situations.

Many systems research efforts are devoted to refinement of various optimization techniques for application to large-scale systems. Large-scale issues with political, economic, technical, social, legal, military, and environmental concerns require significant efforts for design and analysis if there is to be any hope of satisfactory issue resolution from the use of systemic aids. Two MCDT procedures for approaching a large-scale decision situation are multiple objective optimization theory (MOOT) (Besson and Meisel, 1971; Geering, 1971; Roy, 1971; Cochrane and Zeleny, 1973; Cohon and Marks, 1975; Meisel and Payne, 1975; Payne and Polak, 1976; Haimes, 1977; Eliasberg, 1978; Tabak, 1978; Wismer and Chattergy, 1978); and multiple attribute utility theory (MAUT)(Miller, 1967; Raiffa, 1968; MacCrimmon, 1973; Winderfeldt and Fischer, 1973; Keeney and Raiffa, 1976; Edwards, 1977; Farquhar, 1977; Greenwood and Starr, 1977; Huber and Johnson, 1977; Sage, 1977; Keefer, 1978).

Two Approaches to MCDT

The MCDT approaches of MOOT and MAUT can both be utilized in a normative manner, which makes them suited for identifying policies which are aimed at providing solutions to decision situations. These MCDT approaches are comprehensive techniques for complex decision situations.

The MOOT approach is concerned with generating non-dominated solutions to a vector of objective functions. Information from a pre-analysis effort is used to identify a vector of value functions which are then optimized using an appropriate technique. The results of this optimization process generally represent one or more sets of "efficient" solutions from which the DM subsequently chooses the "best optimum". This last action of "best optimum" selection by the DM has not received a proportional amount of attention as has the optimization techniques.

The MAUT approach requires the analyst to elicit preference information from the DM concerning the relative importance of attributes of proposed alternative policies. This information is used to formulate a scalar social choice function which is used to score alternative policies. MAUT seeks to rank alternatives based on the decision makers utility for the outcomes or attributes of those alternatives. MAUT, therefore is intended primarily to be a decision making aid.

While neither approach is entirely new, the development efforts for both approaches have not reached full maturation (Starr and Zeleney, 1977).

4. Systems Engineering Methodology

Our research and application efforts will concentrate on certain steps of the morphological framework of Hall (1969). The logic axis of this structure is divided into seven steps. The first three steps (problem definition, value system design, system synthesis) are a normative approach, as mentioned by Hall (1969), Hill and Warfield (1972), and Sage (1977) to general large-scale systems problems. These steps supply the input description and specification. The system analysis

(and modelling) step develops interrelationships and characteristics of alternatives so as to produce the basis for an impact assessment. Our research is concentrated on the next two steps in the system methodology. Objective functions are optimized with respect to the outcome states in the optimization step. The DM selects the "optimum" policy in the decision making step. These aforementioned steps accomplish parts of the impact assessment and output specification for the system. There is a large variety of optimization and decision making techniques which can be applied, and these different approaches can yield similar results at times (Keeney and Kirkwood, 1977; Gros, 1975; and Tell, 1976).

It is noted that no matter which approach is used for the decision process in a systems effort, the common action oriented phase of producing a transition scenario is of utmost importance to exploit the results of the effort for maximum benefit as discussed by Gibson (1977, 1979). The plan for action step accomplishes the task of optimal policy interpretation and implementation. All the steps would, of course, be iterated in each phase until satisfactory results are obtained.

In a decision situation, the analyst generally seeks a methodological framework for efficient structuring of the factors into a model to aid the DM in selecting and implementing the best alternative course of action. The system engineering morphological framework is a valuable tool for the execution of a systematic analysis of a MCDT situation in coordination with the DM.

5. Dissertation Organization

Our purpose in this research is to motivate and present a more efficient MCDT approach which combines features of the MOOT and MAUT approaches. We concentrate on the optimization/ranking of alternatives and decision making steps of the systems engineering methodology in our efforts. This combined MOOT/MAUT approach is applied to an electronic warfare retrofit problem to produce a design framework.

In Chapter 2, we present MOOT at the theoretical and practice levels as a way of establishing a basis of comparison with MAUT. At the practice level, MOOT is delineated in the systems engineering

framework. In Chapter 3, MAUT is also presented at the theoretical and practice levels. Following the description of MAUT at the practice level in the systems engineering framework, an example is presented from the welfare economics area. This example points out relationships between MOOT and MAUT. Building on the efforts in Chapters 2 and 3, a comparison of MOOT and MAUT is presented in Chapter 4. The combined MOOT/MAUT approach is then presented followed by an example illustrating its use. In Chapter 5, this combined approach is applied to the EWARD problem with the purpose of producing a design framework and set of criteria with which to use in this type of equipment acquisition effort. Chapter 6 presents the summary and conclusions of the dissertation and suggestions for further research.

6. Summary

An introduction to the proposed research was presented in this chapter. An overview of MCDT was presented through an introduction to MOOT and MAUT. The related efforts and contributions of other researchers and authors were enumerated, followed by a general characterization of a decision situation. Finally the application effort of EWARD was introduced, and the remainder of the dissertation was discussed.

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CHAPTER 2

MULTIPLE OBJECTIVE OPTIMIZATION APPROACH TO DECISION SITUATIONS

1. Introduction

Resolution of a complex decision situation can be approached using multiple criteria decision theory (MCDT) (Starr and Zeleny, 1977; MacCrimmon, 1973). Two approaches to MCDT for problem solving are multiple objective optimization theory (MOOT) and multiple attribute utility theory (MAUT). In this effort, MOOT is presented at the theoretical level and at the practical application level for various types of decision situations. In the following work, we discuss MAUT and a combined MOOT/MAUT approach designed to preserve the most advantageous features of each approach. The MOOT discussion is presented from a multiple alternative viewpoint and this allows one to make several observations about dominance and other concepts that, while not entirely original, are not usually considered within the context of multiple objective optimization theory. The MOOT presentation is followed by a simple welfare economics example which illustrates how the MOOT process is applied to a decision situation. This same illustrative example will be used for the MAUT and MOOT/MAUT approaches to facilitate comparison of these approaches to decision situation resolution.

2. MOOT At The Theoretical Level

MOOT is an optimization method for generating optimum solutions for the alternative acts which extremize a vector of performance indices. This vector of performance indicies or objective functions is optimized with respect to each component of the vector performance index. A number of authors discuss computational features of various algorithms for accomplishing this including Cohon and Marks (1975). This optimization of vector objective elements, when combined with a check for dominance, can be used to generate a non-dominated solution set (NDSS). A specific solution alternative is a non-dominated solution (or Pareto optimal solution) if it is not dominated by another solution alternative.

Policy or alternative act A_i dominates policy or alternative A_j if the n vector of performance objectives or attributes for act A_i, which we will call v^i is such that each component of the vector, denoted v_k^i for $k = 1,2,,,n$, is greater than or equal to (with at least one component strictly greater than) the corresponding component of the performance objective vector for act A_j which we denote v_k^j.

Within the context of an attribute hierarchy, the concept of domination is interpreted as follows. We consider the attribute tree of Figure 1. A line is drawn through the hierarchy that separates or divides upper level attributes from lower level attributes. Nodes intersected by these lines are presumably salient attribute nodes for policy comparison. If an alternative policy A_1 has an attribute value at each salient attribute node that is at least as good as the attribute value of another alternative policy A_2 for each corresponding salient node, and if A_1 has one or more attribute values that are better than the corresponding A_2 values, then it will be said that alternative act A_1 dominates A_2 ($A_1 \succ A_2$),and therefore A_2 can be eliminated from further consideration to determine the optimal policy. If an objective hierarchy is decomposed into many levels, it is very likely that there exists a sufficiently low level at which no alternative can be dominated by any of the others* and therefore the decision situation cannot be resolved at this lowest level by alternative elimination using dominance concepts. Therefore, it is convenient to choose a sufficiently high level in the hierarchy of objectives so that alternative policy domination can be established but not so high that there is great axiologic difficulty in determining objectives measures. The set of performance criteria chosen to represent value system design for the decision situation is a conceptual set based on perceptions of the decision maker (DM). Therefore, a permissible set of salient criteria is not limited to a horizontal level set in the objective hierarchy (i.e., level A or B in Figure 1), but can include performance criteria from various levels (e.g., level C). The consequence of the multilevel dominance is

*This makes a, perhaps utopian, assumption that there must be something good in everything.

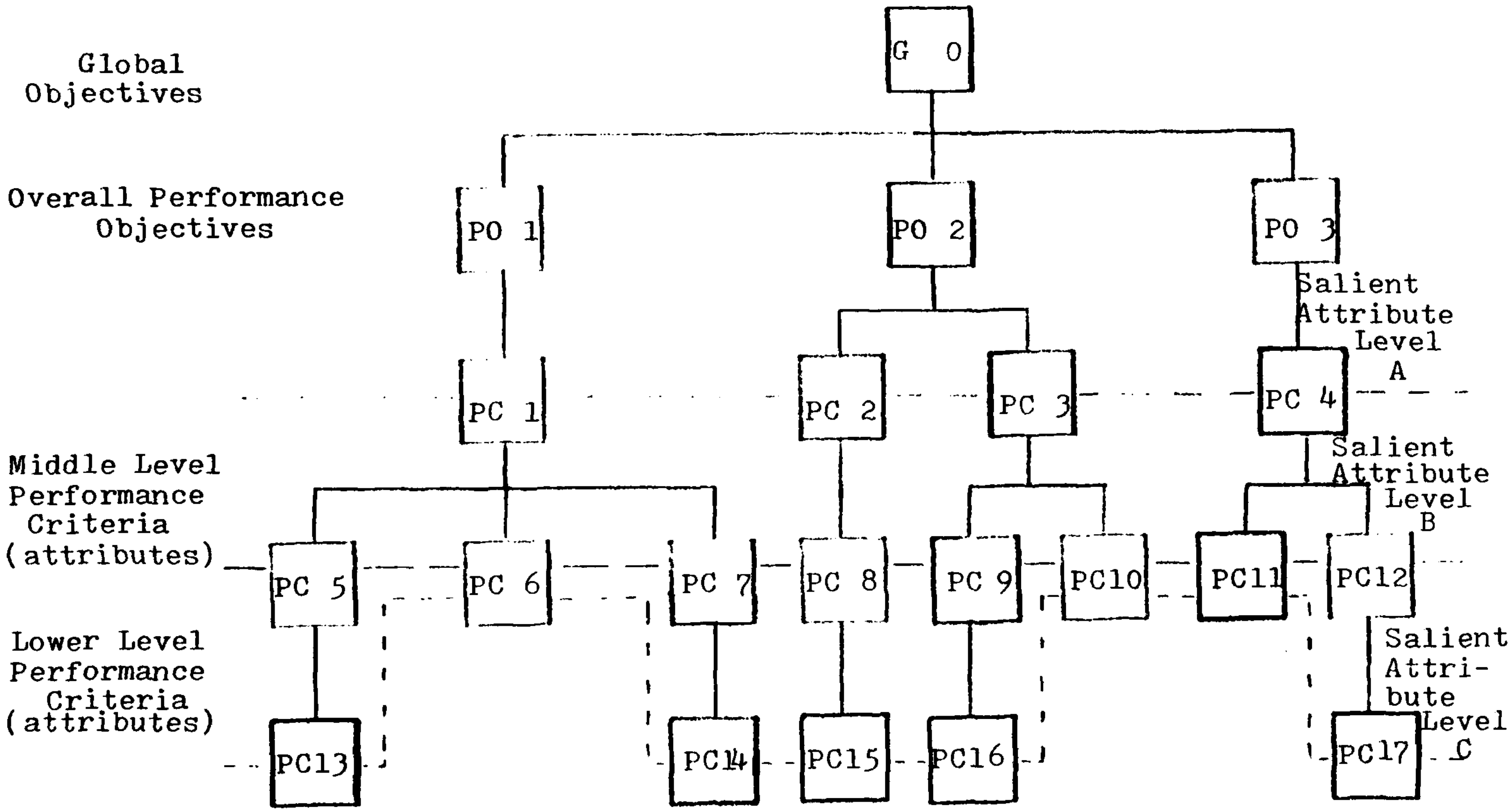


Figure 1. Hierarchial Objective/Attribute Levels For A Large Scale Decision Situation

summarized by the following statement: once an alternative policy is dominated by a second alternative policy, it will remain dominated as one moves to upper levels in the objectives hierarchy. This statement is not necessarily true as one moves from a higher to a lower level. Elimination of all dominated solutions or acts from the initially proposed set of alternatives leaves the NDSS. MOOT has traditionally been concerned with generation of the NDSS or a nondominated frontier of solutions, each of which is optimum in a sense. This NDSS is often called the Pareto optimal set or Pareto frontier. This discussion asserts an often overlooked but useful fact that the acts contained in the NDSS will be very much a function of the hierarchial level at which policy comparisons are made. This is a very useful concept for successful use of MOOT concepts.

2.1 MOOT - NON-STOCHASTIC/TIME INVARIANT CASE

The theoretical multiple objective optimization problem for the deterministic or non-stochastic and time invariant or static case is posed as a vector criterion optimization problem. Consider a policy vector, a, and a policy outcome state or event vector, x. An objective vector is described by $j(x,a) = J$ where the components are element objective functions. The constraints are described by a vector equality set $f(x,a) = 0$ and an inequality set $g(x,a) \leq 0$. The system relationship between a and x along with representations or characterizations of forbidden regions in policy act - outcome event space make up constraint components. The problem is to maximize the performance objective components subject to the system equations and constraints. This problem is represented mathematically as

$$\text{maximize } J = \text{maximize } j(x,a) \tag{1}$$

$$\text{subject to: } f(x,a) = 0 \tag{2}$$

$$g(x,a) \leq 0 \tag{3}$$

Since j is a vector, special attention must be directed at the meaning of simultaneous optimization of the components of the vector. This optimization may be accomplished in a "constraint method," "attainment method," or goal programming" format (Cohon and Marks, 1975) where a

specific objective function element (j_i) is optimized while holding the other cost function elements ($j_{\bar{i}}$) equal to a constant ($j_{\bar{i}} = \overline{j_{\bar{i}}}$). The formulation now becomes

$$\text{maximize } j_i(x,a) \tag{4}$$

$$\text{subject to: } j_{\bar{i}} = \overline{j_{\bar{i}}} \tag{5}$$

$$f(x,a) = 0 \tag{6}$$

$$g(x,a) \leq 0 \tag{7}$$

where $j_{\bar{i}}$ is a vector of all objective elements except j_i. The necessary conditions called the Kuhn-Tucker conditions, (Taha, 1976) for $\hat{a}$ to be a stationary point for each maximization are:

$$g(\hat{x},\hat{a}) \leq 0 \tag{8}$$

$$f(\hat{x},\hat{a}) = 0 \tag{9}$$

$$j_{\bar{i}} = \overline{j}_i \tag{10}$$

$$\frac{\partial j_i(\hat{x},\hat{a})}{\partial \hat{a}} - \frac{\partial j_{\bar{i}}(\hat{x},\hat{a})}{\partial \hat{a}}\lambda - \frac{\partial g(\hat{x},\hat{a})}{\partial \hat{a}}\theta - \frac{\partial f(\hat{x},\hat{a})}{\partial \hat{a}}\gamma = 0 \tag{11}$$

$$\lambda^T(\overline{j_{\bar{i}}} - j_{\bar{i}}) = 0 \tag{12}$$

$$\gamma^T(f(\hat{x},\hat{a})) = 0 \tag{13}$$

$$\theta^T(g(\hat{x},\hat{a})) = 0 \tag{14}$$

$$\lambda^T,\ \gamma^T,\ \theta^T \geq 0 \tag{15}$$

where $\hat{a}$ is the optimal policy and $\hat{x}$ is the value of the outcome states for the optimal policy, λ is a Lagrange multiplier vector for the $j_{\bar{i}} = \overline{j}_i$ set of constraints, γ is a Lagrange multiplier vector for the $f(x,a) = 0$ set of constraints, and θ is the Lagrange multiplier vector for the $g(a,x) \leq 0$ set of constraints. This set of equations is solved many times for different feasible values of $j_{\bar{i}}$ to generate a set of solutions. Calculations from equations 4, 5, 6, and 7 will define the limits of $j_{\bar{i}}$ to insure that a specific $\overline{j_{\bar{i}}}$ is feasible. These equations are solved for the values of $\hat{a}$ and $\hat{x}$ for each $\overline{j_{\bar{i}}}$. Each time $j_{\bar{i}}$ takes on a specific value, the analyst uses the optimization process to find one feasible solution (if the constraints are not violated) to the original

maximization of the j(x,a) formulation. The values of $j_{\bar{I}}$ can be allowed to vary sequentially over their ranges, or these values can be set equal to a set number of attainment levels. Since usually one or more of the objectives are non-commensurate and conflicting in nature, no individual form of a policy from the set of feasible policies will usually allow a global optimum in the general case. Instead, an elimination by dominance exercise selects those feasible policies which are non-dominated thereby forming the NDSS. Figure 2 illustrates the NDSS for four hypothetical policies with respect to two attributes or value functions of the DM. Assuming more is preferred to less with respect to the attributes, policy 1 in Figure 2a is dominated by policy 2. Figure 2b shows the results of two policies where neither is globally dominant. The overall NDSS is comprised of alternative forms of policy 3 and policy 4. In the latter case, the selection of the optional policy can be accomplished by the introduction of value judgments of the DM. Traditionally, however, the DM's value judgments have not been incorporated explicitly into MOOT and it is a goal of this effort to render MOOT approaches more useful at the practice level by doing this.

2.2 MOOT - Time Invariant/Uncertain Outcomes

Very often, elements surrounding the decision situation are not known with certainty. This uncertainty can generally be characterized by probability distributions for outcome states associated with different possible events. The degree of difficulty in obtaining MOOT problem solutions generally increases substantially by introducing probabilistic notions. The general formulation is

$$\max \quad J = E\{\, j(x,a,\xi)\} \tag{16}$$

$$\text{subject to} \quad f(x,a,\omega) = 0 \tag{17}$$

$$g(x,a,\nu) \leq 0 \tag{18}$$

where ξ, ω, and ν are realizations of random processes, and E denotes expectation. In the general case, for a continuous state representation, the expectation can be conditioned upon an observation. A form which is more mathematically tractable is

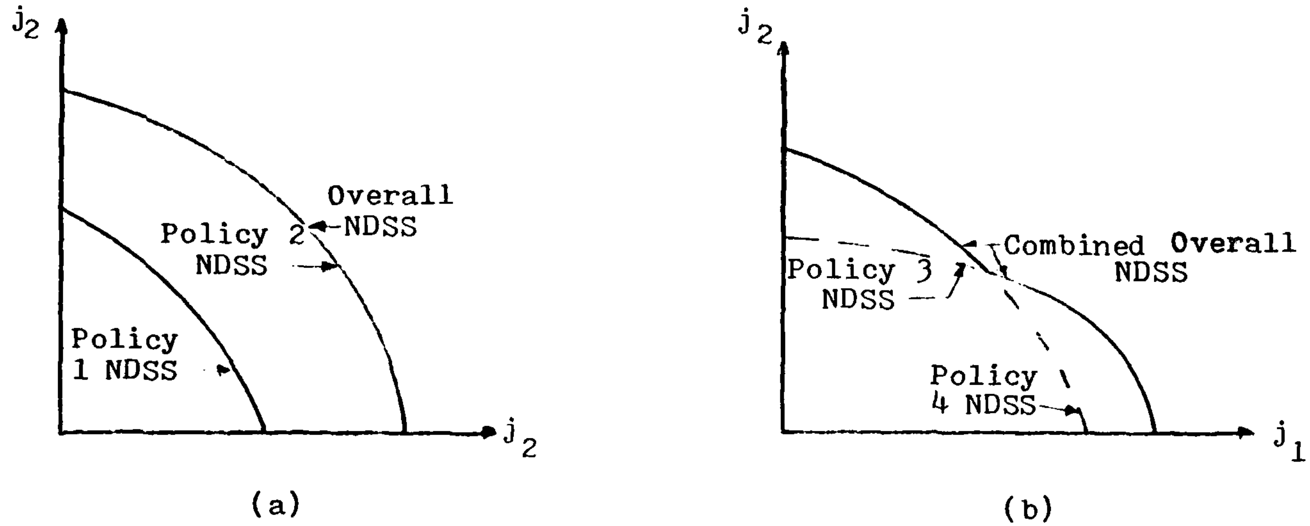


Figure 2. Non-Dominated Solution Sets (Attribute Vector $j^T = (j_1, j_2)$)

$$\max \qquad J = E\{\Phi(x,a)\} \tag{19}$$

$$\text{subject to} \qquad f(x,a) + \omega = 0 \tag{20}$$

$$g(x,a) + \nu = 0 \tag{21}$$

Optimization is accomplished after transforming these formulations into the general form of equations 4, 5, 6, and 7 through a constraint technique or goal programming.

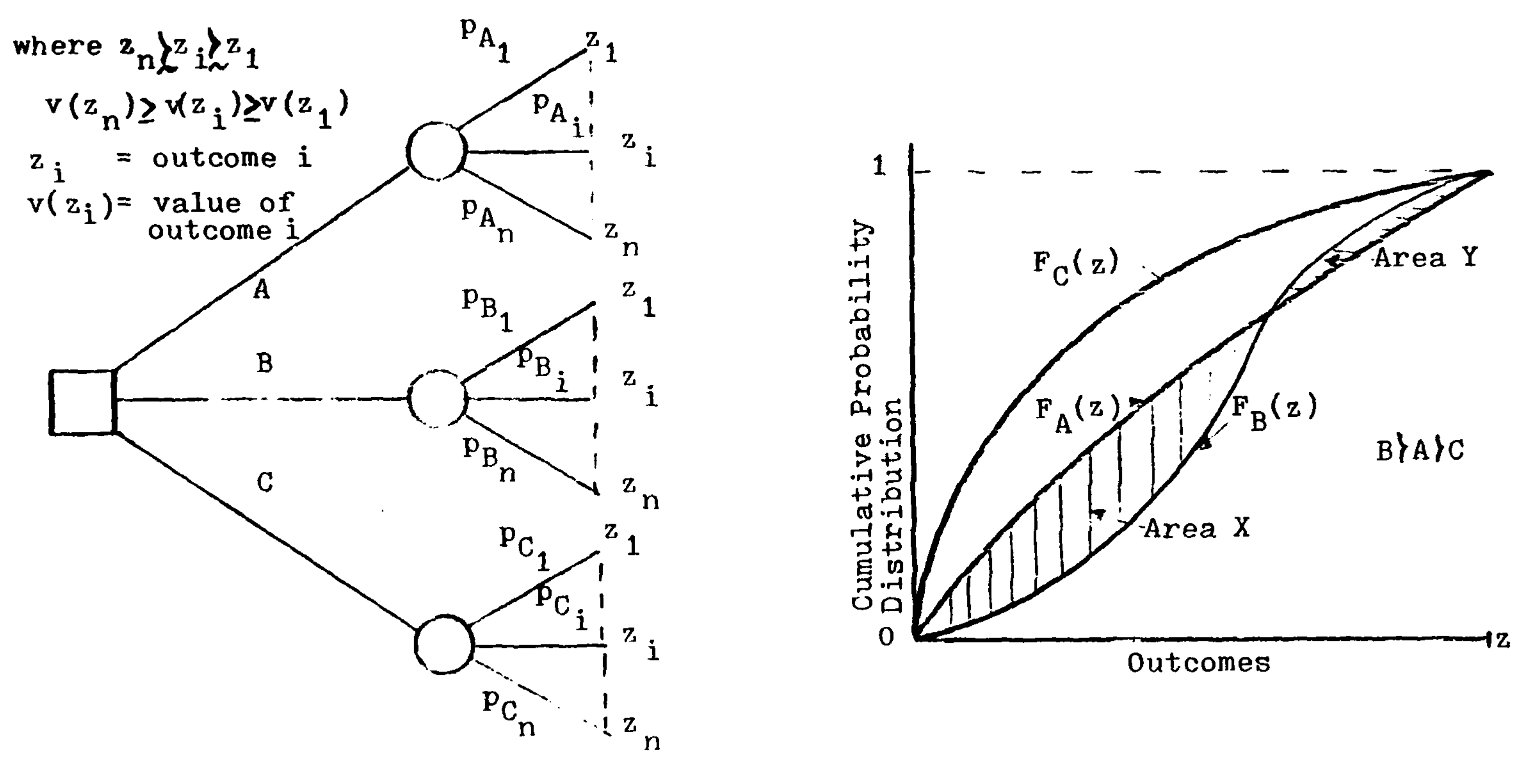
Utilizing the concept of stochastic dominance, the analyst may be able to rapidly discern the optimal policy by comparing the cumulative distribution functions for the various alternatives. Stochastic dominance is said to occur if the expected value of an alternative is greater than that of another over a whole class of value functions. For a decision situation which is represented by the decision tree of Figure 3a, assume that the set of possible outcomes is the same for all alternatives and that the outcomes are arranged in order of increasing goodness so that the value function for this set of outcomes is a non-decreasing function. If one plots the cumulative probability distribution functions (CDF) of the alternatives for the outcomes, $F_i(z)$, first degree stochastic dominance (FDSD) is easy to recognize. Alternative i will dominate alternative j if the CDF

$$F_j(z) \geq F_i(z) \tag{22}$$

for all outcomes. In practice, one can easily recognize any CDF which lies completely to the left of another alternative's CDF without intersecting it. Figure 3b illustrates that alternative C is dominated (in the sense of FDSD) by both alternatives A and B. If the CDF curves intersect, FDSD will be inconclusive, but second degree stochastic dominance (SDSD) may further screen the set of options. SDSD requires the DM to be risk averse over the outputs (the utility function must be concave). SDSD states that alternative i will dominate j if

$$\int_{\text{Outcomes}} (F_j(y) - F_i(y))dy \geq 0 \tag{23}$$

for all z. That is, the area under the $F_j(z)$ should not be less than the area under $F_i(z)$. In Figure 3b, it is clear that area X > area Y and therefore alternative B dominates alternative A in the sense of SDSD.



(a) (b)

Figure 3. Stochastic Dominance

As shown, stochastic dominance may be helpful in MOOT for screening alternatives in certain decision situations. The theory for the basis of FDSD and SDSD is discussed by Hancock and Levy (1969), Hadar and Russell (1969), and Bunn (1978).

It is noted that Pareto optimality may not satisfy requirements for identical levels of preference for consequences among individuals in a decision situation. Here an identical level of preference for two individuals is defined as a position for attributes which is established by an identical trade-off ratio for the two individuals (e.g., consider an outcome state which is described by a_1 units of attribute A_1; this outcome state value would constitute an identical level of preference for two individuals if each would be willing to trade-off exactly b_1 units of another attribute B_1 for a_1 units of A_1. In a decision situation, it is possible that based on identical levels of preference, the optimal policy is not necessarily a member of the NDSS. This is of considerable concern in situations which involve uncertainty and multiple DMs. Consider the example of three alternatives with the following possible outcome states:

alternative 1: $p = .5$ — $v_1 = .9;\ v_2 = .1$ $\qquad E(v_1) = .5$
$\qquad\qquad\qquad p = .5$ — $v_1 = .1;\ v_2 = .9$ $\qquad E(v_2) = .5$

alternative 2: $p = 1$ $\qquad v_1 = .3;\ v_2 = .6$ $\qquad E(v_1) = .3;\ E(v_2) = .6$

alternative 3: $p = 1$ $\qquad v_1 = .45;\ v_2 = .45$ $\qquad E(v_1) = E(v_2) = .45$

where v_i = the level of preference or value of an outcome by individual i and $E(v_i)$ is the expected value of the outcome of the specific alternative for individual i. Based on an expected value criteria, alternatives 1 and 2 are non-dominated (clearly alternative 1 dominates alternative 3). In alternative 1, depending on which event occurs, either individual 1 is going to receive a more desirable outcome and individual 2 a less desirable outcome or vice-versa. In alternative 2, clearly individual 2 is going to receive a more desirable outcome. Alternative 3 is the only alternative where both individuals are going to receive the same outcome based on an identical level of preference, yet it is not in the NDSS. This illustrates that any choice from the NDSS may not take

into account identical levels of preference and therefore not be optimal with respect to these equity considerations. Therefore, one is warned that if equality with respect to levels of preference for consequences is of paramount importance, MOOT does not insure one of selecting the optimal policy.

2.3 MOOT - Non-Stochastic/Time Varying Case

In the previous discussions of MOOT, static or single stage decision problems were considered. MOOT is capable of incorporating dynamic elements, that is considerations of events evolving over time. In reality many decision situations should be modelled in this way. The optimization formulations now can be described in terms of the exogenous variable, time (t). For instance, the vector objective function can now take the form of

$$J = \int_{t_{initial}}^{t_{final}} \phi[x(t), a(t), t]dt \qquad (24)$$

where this objective function gives an indication of policy performance as a function of time, and the constraints may have the form

$$\dot{x} = f[x(t), a(t), t]dt \qquad (25)$$

$$x(t_{initial}) = x_o \qquad (26)$$

where $t_{initial}$ is fixed and t_{final} may be fixed or variable (Sage and White, 1977). This formulation is valuable to policy determination because of dependency of the state elements in many decision situations on time. Dynamic optimization tools, which are utilized to determine the optimal policy for a dynamic MOOT formulation, are applied in a similar manner as the static case in order to identify and optimize the non-dominated policies. The effort required for dynamic optimization with multiple objectives is substantially greater than that associated with the static case. The system dynamics can be formulated in continuous or discrete time representations.

2.4 MOOT - Outcome Uncertainty/Time Varying Case

A decision situation which involves uncertainty associated with event outcomes and where event outcomes evolve over time presents the greatest level of complexity for a MOOT formulation. The problem formulation can now include descriptions of the event outcomes in terms of probability distributions and evolution over time. The solution form on which to base the NDSS is an optimal strategy which can be produced using stochastic optimal control theoretic techniques. The probabilistic/dynamic version of the vector criteria problem has seen the least development of the MOOT areas. Some of the few efforts in this area are in Markov decision processes.

Regardless of which of the representations described in the preceding sections is used, the purpose of MOOT is to allow the analyst to optimize with respect to the multiple criteria and to then form the NDSS by eliminating the dominated solution. This non-dominated solution set is then presented to the decision maker for selection of the most preferred solution.

3. A Multiple Objective Optimization Process

Multiple objective optimization theory is currently the focus of considerable research, and it is likely that there is disagreement among practitioners and researchers concerning the bounds of this approach and the specific process to use in aiding resolution of a decision situation. While it is basically an algorithm for optimization for later decision making, many practitioners of multiple objective decision making include all steps of the systems engineering approach within the MOOT process algorithm at the application level, as does this description. The MOOT process is centered around optimization and subsequent formation of the NDSS, although at the practice level, a MAUT or some other value aggregation technique is needed for the selection of an optimum form of the NDSS for the actual decision step. The addition of a formal decision making step, which is usually not included in the traditional descriptions of MOOT at any level, is required in order to select the optimal

policy from the NDSS.

3.1 Description of the Deterministic/Static MOOT Process

The following is a description of a deterministic/static MOOT process at the application level. Since the deterministic/static case is a general form of MOOT, much of this description is also applicable to the other cases of MOOT. While the cases which don't consider time or its evolution explicitly are generally called static, these cases often consider time in an implicit and wholistic way. That is, these cases are concerned with the situation at a terminal time and not necessarily on the situation at an intermediate time. It is intended that this process description will bring out the characteristics needed for a subsequent comparison with MAUT. Multiple Objective Optimization Theory (MOOT) applied at the practice level uses mathematical optimization of multiple criteria to facilitate the identification of an optimal alternative action as a solution to a decision situation. MOOT incorporates multiple criteria into the optimization step, thereby requiring optimization with respect to this vector of performance objectives. A comprehensive pre-analysis effort to determine input specification is the initial phase of the MOOT process. The MOOT process at the application level includes the traditional systems analysis/modelling and mathematical optimization steps of impact assessment, and the policy selection/decision making step of output specification.

The usual techniques utilized in the impact assessment of the MOOT process are from the area of mathematical programming.

The use of the MOOT process for a practical application is predicated on the following assumptions:

1. The relationships governing the system described by the decision situation can be expressed in mathematical equations.

2. The attributes used to measure performance of the alternative actions should be established as preferentially independent (to insure that trade-offs can eventually be made to identify the optimal policy). This PI assumption is not needed to determine the NDSS but is generally needed for final trade-off analysis and alternative selection.

Traditionally, the relationship among the objectives has not been considered in MOOT. Generally, the objectives have been assumed independent essentially by default.

3. The DM will provide preference information which allows the formation of a scalar choice function for combining the objectives and subsequent selection of the optimal policy from a NDSS.

Output Results of the MOOT Process

Application of the MOOT process generally results in a set of mathematical equations which describe how the states evolve or are impacted by alternative acts and any constraints on this evolution. From a specified vector cost function, a set of non-dominated solutions which result from optimization with respect to the vector criteria, a quantification of preferences of the DM, an identification of the trade-offs present in the situation, an identification of the optimal course of action, and an indication of the costs and benefits of implementing the optimal course of action result. Additionally, as a by product of determining the optimal policy, information is usually generated which indicates the elements to which the solution is most sensitive.

MOOT Process Algorithm

Several steps are involved in the application of MOOT, and these steps are accomplished in an iterative manner.

a. A pre-analysis phase is directed toward defining the scope and boundary, and identifying the elements of the decision situation. Statements of the objectives, alternate actions, and constraints are developed for the decision situation. The input specification portion of a systems effort yields this information. Preferential independence should be established among the attributes to insure that there is a quantitative understandable basis for trading-off individual units of the attributes. If PI among attributes cannot be established, then analytic trade-offs are either not possible or at best are very difficult, since the DM views all attributes as interrelated and interdependent. The only method for decision making in this case is appeal

to the DM's intuition. If PI is established, this only allows proceeding with the multiple criteria approach. Later in the modelling step, further elicitation concerning the attributes is required such as investigation of the relationship of attributes with respect to weak difference independence, mutual difference independence, mutual preferential independence, etc., in order to establish the form (linear additive, multiplicative, etc.) of the quantitative value function (Dyer and Sarin, 1979; Keeney and Raiffa, 1976). Additionally, one must then evaluate certain levels of the attributes with the DM in order to establish the scaling parameters in the value function. The selection of an appropriate level to approach a decision situation is paramount to insure a tractable problem which has contextural integrity.

b. A systems analysis/modelling phase is used for the construction of a precise mathematical model. The elements which were identified in the pre-analysis phase are organized into a set of equations and related mathematical expressions which adequately describe the decision situation. The objectives and goals of the DM are included in the model. A set of value functions (objective or cost functions) of the DM are used as vector criteria to judge the goodness of alternative decisions, (e.g., max J_1 = maximize profit (P), and min J_2 = minimize time spent (t)) and a set of alternative actions or alternative systems each composed of a set of decision variables and process equations. The decision variables represent specific activities (e.g., allocation of two resources in amounts a_1 and a_2) whose optimum value is to be determined in each alternative policy. The objective functions are mathematical functions of the state and occasionally the decision variables. These objective functions are optimized (maximized or minimized) individually subject to a set of restrictions or constraints on the state and decision variables. These constraints are expressed mathematically as inequalities or equalities.

c. Optimization of the objective functions is accomplished, generally with the aid of a digital computer. An objective function is optimized within the bounds of the constraints while all other objective functions are held at or near some prescribed attainment level. The

other objective functions become in effect additional constraints (e.g., max J_1, S.T. $J_2 = \overline{J}_2$; where $\overline{J}_2$ is set at various values). While a set of solutions is generated from optimization, an optimization management routine (Beale and Cook, 1978) utilizing elimination by domination checks each solution to see if it is a non-dominated solution. These non-dominated forms of the alternative actions represent the best the various alternative actions can do with respect to the vector of criteria.

A caveat is warranted concerning the permanent elimination of dominated alternatives. A dominated alternative has the potential of being ranked the second best. This is of particular significance if it is determined later that the optimal policy selected from the NDSS is not implementable. In this case, one must avoid identifying the next best alternative from the NDSS as the new optimal without first checking all feasible alternatives to see if any new members of the NDSS are produced. It is conceivable that a previously dominated alternative will be in the NDSS due to the elimination of a non-implementable previous optimal alternative. It is generally possible to identify this new alternative as optimal through the use of a scalar scoring function (Appendix B). Eliminating alternatives which are dominated by only one or a few other alternatives can cause only the best and the worst of the alternatives to be scored with a scalar choice function in the succeeding policy selection step. Transitivity of preferences can cause this comparison of alternative extremes. The dominance relation is transitive and elimination of dominated alternatives is justified because of this transitivity. For instance, consider transitive multiattribute alternatives ($\bar{a}, \bar{b}, \bar{c}$, and $\bar{d}$), such that $\bar{a}$ dominates $\bar{b}$ ($\bar{a} \succ \bar{b}$), and $\bar{b}$ dominates $\bar{c}$ ($\bar{b} \succ \bar{c}$). Transitivity would allow us to eliminate alternatives $\bar{b}$ and $\bar{c}$ thus compare only the $\bar{a}$ and $\bar{d}$ with a scoring function. Depending on the scoring function, it is possible for either $\bar{b}$ or $\bar{c}$ to have a larger score than $\bar{d}$ and in that case we would not be considering the top two choices, but the first and last choice. Intuitively, it is more appealing to trade-off the top ranked alternatives instead of the top and bottom ranked alternatives. Therefore it is recommended that an alternative not be eliminated only on the basis of a single dominance by another alternative. Instead it is recommended to require that all or

a majority of alternatives dominate a given alternative before it can be safely eliminated.

When a group of DMs are involved in the decision situation, the Pareto criterion may be a very important tool to eliminate the inferior alternatives since interpersonal comparison of preference is not required to form the DNSS.

d. Policy selection is accomplished when the DM chooses the most preferred solution from the NDSS. There are various ways to do this, such as using an indifference function, and a surrogate worth trade-offs (Haimes, Hall, and Freedman, 1975). All of these structure the DM's preferences and in effect form a scalar social choice function (SCF), composed of the objective functions used in the optimization phase (e.g., $J = f(J_1, J_2)$).

In the general case, preference information is elicited from the DM (Dyer and Sarin, 1979) in order to generate the functional form as well as the weights (scaling constants, w_i) of the SCF (e.g., $J = w_1J_1 + w_2J_2$). Once the SCF is constructed, it is used to identify the single optimum course of action from the NDSS. This reduction of the NDSS is, in effect, a post-optimization ranking exercise which often requires a digital computer utilizing adaptive search procedures (Cohon and Marks, 1975) and exhaustive comparison (Anyiwo and White, 1978).

An output from MOOT, which may be useful to the analyst in presenting the alternatives to the DM, is the NDSS. This may be of value in the general case and is of particular value if there are only two objective functions because a graphical display of the NDSS is now possible.

e. Analysis of the results is usually conducted to evaluate the robustness of the solution to variation in parameters. Critical parameters are identified, the sensitivity of the solution to variations in these parameters is accomplished.

f. An algorithm for the MOOT process is shown in the DELTA chart of Figure 4. The policy selected in the decision making step is used as the input into the planning for action step. The steps of system analysis/modelling optimization and decision making as shown in Figure 5

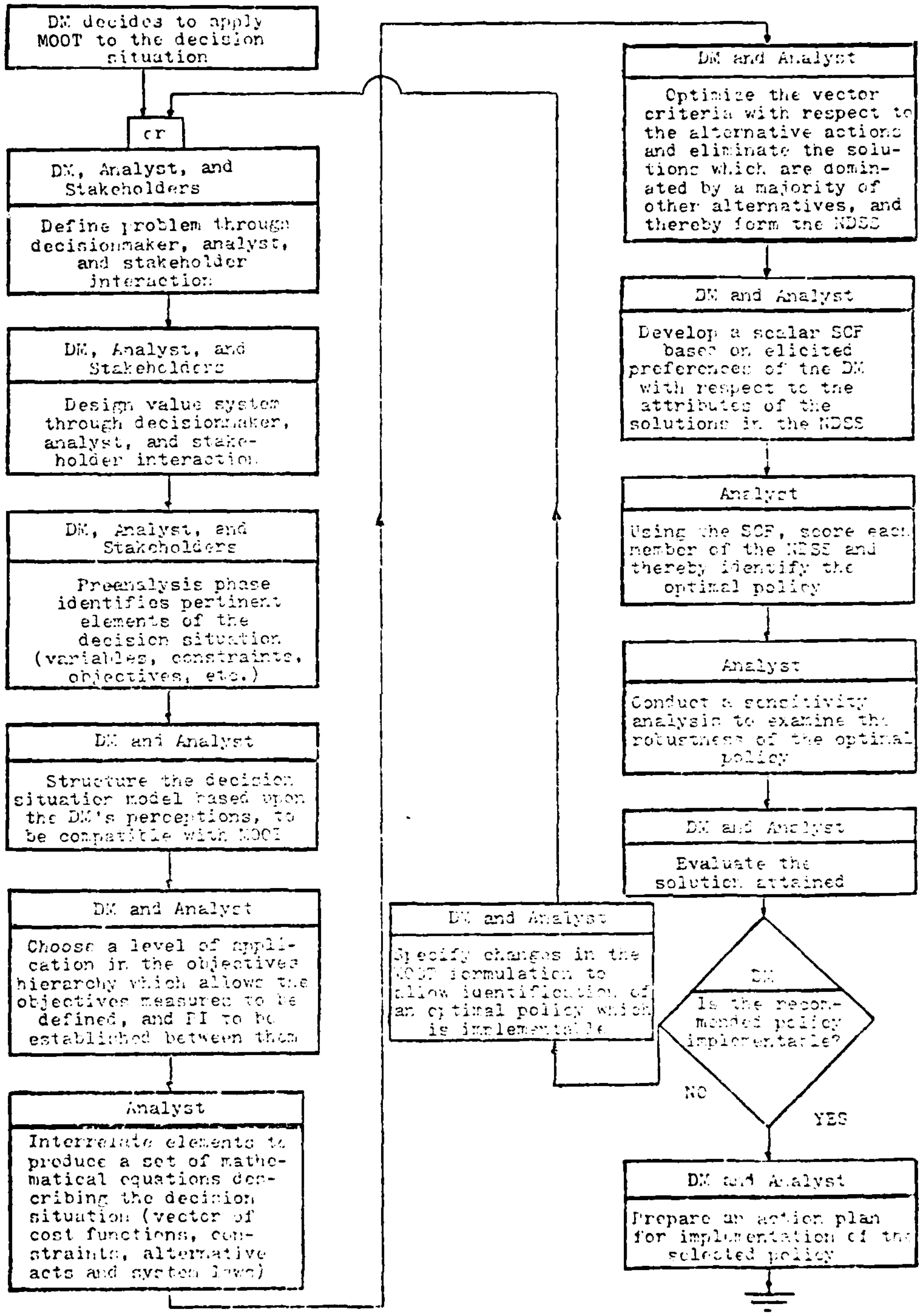


Figure 4. DELTA Chart For The Deterministic/Static Case Of MOOT

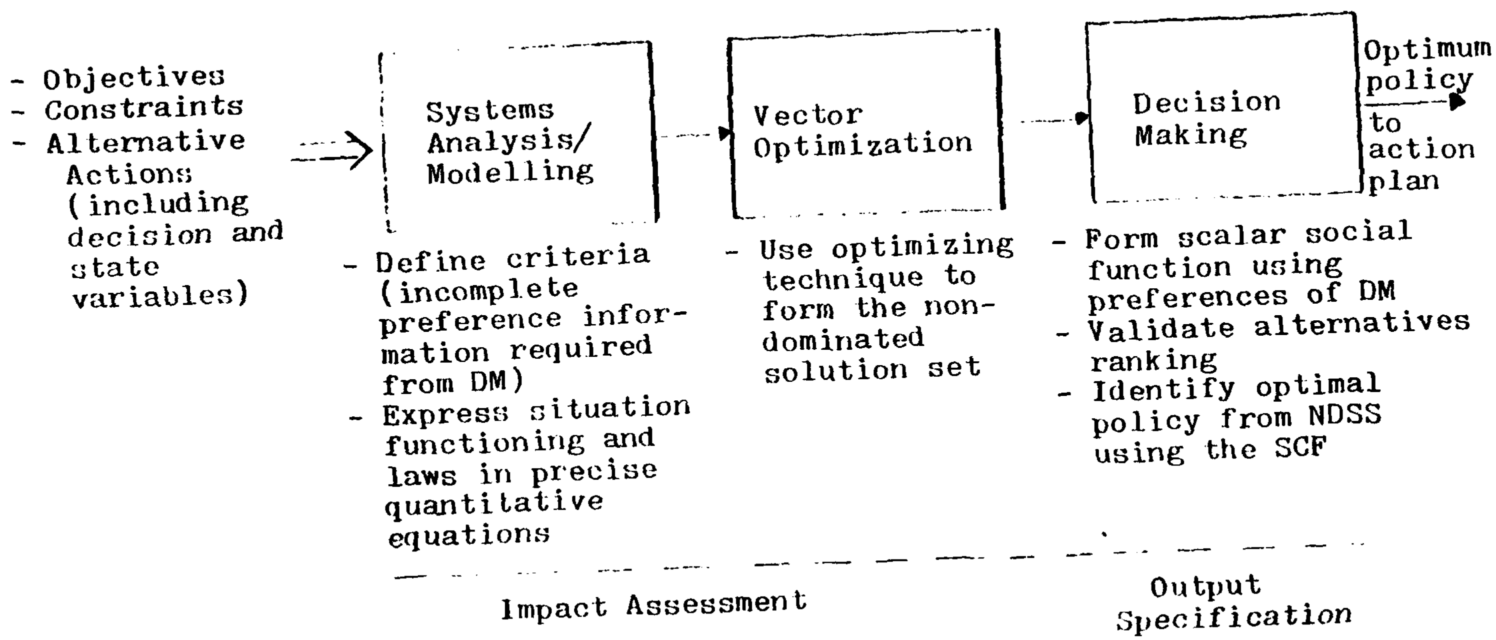


Figure 5. MOOT Formulation

will be in areas of concentration in the subsequent comparison of MOOT and MAUT processes.

Appropriate Conditions for Use of the MOOT Process

The application of the MOOT process requires a set of necessary conditions which include the following:

- A set of alternative actions and criteria to judge the relative goodness of these alternatives quantitatively must exist and must be expressible in mathematical or numerical form.
- The decision situation functional laws, constraints, and impacts of pertinent variables must be expressible in mathematical form.
- The DM must provide information to aid in structuring the situation and establishing performance measures and preferential independence between them.
- The DM must express criterion weights associated with the attributes in order to form a SCF when this is needed for post-optimization policy selection.

MOOT is appropriate for selecting the best alternative policy by mathematical optimization techniques when the complexity or size of the decision situation makes intuitive solutions extremely difficult to obtain. MOOT can be used for situations with linear and non-linear relations between policy and state variables. MOOT is particularly adept at handling physically motivated criteria based on precise mathematical relations because of the refinement and tuning of alternative policies available through the optimization techniques. In practice, MOOT is utilized to select the best of a set of proposed policies where the decision situation is composed of quantifiable parts. This makes MOOT particularly valuable in hardware oriented decision situations, such as in determining optimum policy parameters in subsystem equipment design. The MOOT approach is well suited for application to public and private areas typically involving resource allocations.

Input Requirement for the MOOT Process

Information in quantitative form needs to be available for

interrelating the salient elements of the decision situation. These elements include: a set of stated objectives or goals which are preferentially independent and by which the relative goodness of each alternative solution can be judged; a set of alternative policies; and a set of any restrictive constraints. These elements are organized into a set of mathematical equations through a structuring and modelling effort.

Essential personnel required for the MOOT process are the DMs responsible for the decision situations and an analyst familiar with MOOT. Generally other stakeholders such as advisors and experts are also involved to some degree in MOOT in the modelling effort.

An essential facility for the MOOT process is a digital computer. Access to the computer with rapid computation and display capability facilitates management and presentation of data for the stakeholders, as well as optimization in the multiobjective realm. Additionally, graphical display materials (e.g., video CRT and CRT projectors, view-graph projector, chalkboard, etc.) will aid in interaction between the analyst, DM, and stakeholders.

A set of questions which should be asked at the conclusion of the MOOT process to check the integrity of the process and the resulting optimal policy are as follows: Does the mathematical model of MOOT satisfactorily mimic the decision situation so as to give the DM confidence in the indicated results? Does the DM understand the model sufficiently to give he or she confidence in the results? Are the results consistent with the objectives and goals of the DM and organization responsible for the decision situation? Is the policy produced by MOOT implementable? Would simple intuitive judgment lead to as good a solution to the decision situation? An affirmative answer to this last question would indicate that MOOT is not needed to generate a solution to this decision situation in the future.

Computer programs for optimizing the individual criteria in the optimization process are available on most large computer systems (e.g., MPOS, CDC optimization pack, etc.) for at least the deterministic/static case. The optimization management programs which sequentially accomplish (manage the routines which optimize the individual criterion) the

optimization of the vector of criteria and subsequently check for non-dominated solutions are not as common. Some specific examples of optimization management programs are listed by Beale and Cook (1978); Tabak et al., (1978); and Steuer (1977).

Various applications of MOOT have been accomplished, such as an aircraft control systems design (Tabak, et al., 1978), the composite design of a Merchant Marine Fleet (Everett, et al., 1972), a simulation of aircraft performance (Beale and Cook, 1978), and the selection of an optimal policy for water resource management (Haimes, 1977).

3.2 Modifications To The MOOT Process Due To Probabilistic And Dynamic Considerations

As mentioned in the theoretical descriptions, when either dynamic or probabilistic elements are included in the MOOT formulation, the complexity level increases. The primary steps which are effected are the systems modelling/analysis and optimization steps. One must take care to insure that the model formulation of the multiple objectives and constraints accurately incorporates the dynamic flow and probabilistic elements of the decision situation. This generally requires the use of differential equations (difference equations in the discrete time representation) to represent time flows, a probability transition mechanism in the finite state representation, and the use of the expectation operator to tractably work with the probabilistic elements. The optimization step may change because an optimal sequential set of policies or strategy may be required. This optimal strategy may occasionally take on the form of a randomized strategy (in period 1, implement act 1 with .3 probability, or act 2 with .7 probability), but because of the difficulty implementing this strategy, the DM may prefer a deterministic strategy.

4. An Example of MOOT

As an illustration of MOOT, and as a way of setting up a basis for comparison of the conditions required for optimization in the MOOT

process and MAUT process, we will consider an example from the area of welfare economics. Welfare economics is that branch of economics which deals with the distribution and consumption of resources for the public good as opposed to the individual good. The general objective of welfare economic analysis is the evaluation of economic alternatives and redistribution of economic resources for maximum societal benefits. An allocation of resources is Pareto optimal when no other reallocation of production and distribution will increase the economic satisfaction of any one individual without decreasing the satisfaction level of others in society.

Consider the following formulation of society's economic problem. For concreteness, we shall assume a simple closed economy with two consumers. We will assume that the economy is endowed with two factors, capital C and labor L. There are two outputs from production, a (clothing) and b (food). There are fixed endowments of labor and capital $\overline{C}$ and $\overline{L}$ described by

$$\overline{C} = C_a + C_b \tag{27}$$

$$\overline{L} = L_a + L_b \tag{28}$$

where $\overline{C}$ and $\overline{L}$ are the maximum levels of factor supplies, C_a and C_b are the amount of capital allocated for producing a and b respectively, and L_a and L_b indicates the amount of labor which is allocated for producing a and b respectively. The production functions which determine the amount of clothing and food produced are given by

$$a = f_i\ (C_a,\ L_a) \tag{29}$$

$$b = f_j\ (C_b,\ L_b) \tag{30}$$

Two alternative actions (A_1, A_2), which are each composed of different production functions, have been defined. Each alternative action is composed of the four policy variables C_a, C_b, L_a, L_b. In this example, not only will the optimum alternative which maximizes the utility of the two consumers be identified, but also the best levels of the policy variables for the optimum will be found.

We assume that all agricultural and clothing production will be

distributed between the two consumers as needed to maximize their happiness. Thus we have

$$a = a_1 + a_2 \tag{31}$$

$$b = b_1 + b_2 \tag{32}$$

where a_1 and a_2 are the amount of clothes allocated to consumers 1 and 2 respectively, and b_1 and b_2 are the amount of food allocated to consumers 1 and 2 respectively. The objective of each consumer is to maximize his or her utility.

We assume that the utility of each consumer depends directly on the quantity of products consumed. The utility functions for the consumers are assumed to be the isotone functions

$$u_1 = g_1 (a_1, b_1) \tag{33}$$

$$u_2 = g_2 (a_2, b_2) \tag{34}$$

The vector of objectives functions is defined as

$$\text{Max } J = \text{Max } (u_1, u_2) \tag{35}$$

The optimization step commences with the optimization of one objective function while holding the value of the other objective function constant.

Now we can pose the above problem as

$$\text{Max} \quad u_1 \tag{36}$$

$$\text{subject to} \quad u_2 = \bar{u}_2 \tag{37}$$

$$\bar{C} = C_a + C_b \tag{38}$$

$$\bar{L} = L_a + L_b \tag{39}$$

$$a = f_i(C_a, L_a) = a_1 + a_2 \tag{40}$$

$$b = f_j(C_b, L_b) = b_1 + b_2 \tag{41}$$

where $\bar{u}_2$ is a specific level of utility for consumer 2. Now adjoining the constraints to u_1 with Lagrange multipliers, optimizing to obtain the necessary conditions, and manipulating those conditions to eliminate the Lagrange multipliers yields three equations:

$$\frac{\frac{\partial u_1}{\partial a_1}}{\frac{\partial u_1}{\partial b_1}} = \frac{\frac{\partial u_2}{\partial a_2}}{\frac{\partial u_2}{\partial b_2}} \qquad 42$$

where $\frac{\partial u_1}{\partial a_1}$ and $\frac{\partial u_1}{\partial b_1}$ indicates the marginal utility of consumer 1 for products a and b respectively, and $\frac{\partial u_2}{\partial a_2}$ and $\frac{\partial u_2}{\partial b_2}$ indicate the marginal utility for consumer 2 for products a and b respectively;

$$\frac{\frac{\partial a}{\partial L}}{\frac{\partial a}{\partial C}} = \frac{\frac{\partial b}{\partial L}}{\frac{\partial b}{\partial C}} \qquad 43$$

where $\frac{\partial a}{\partial L}$ and $\frac{\partial a}{\partial C}$ are the marginal products of labor and capital respectively in industry a, and $\frac{\partial b}{\partial L}$ and $\frac{\partial b}{\partial C}$ are the marginal products of labor and capital respectively for industry b;

$$\frac{\frac{\partial u_1}{\partial a_1}}{\frac{\partial u_1}{\partial b_1}} = \frac{\frac{\partial b}{\partial C}}{\frac{\partial a}{\partial C}} \qquad 44$$

where the three equations represent efficient consumption, production, and product mix respectively. The optimization problem is optimized many times for various values of $\overline{u}_2$ (effectively this is allowing the Lagrange multiplier to change values). Iterating the optimization procedure with each alternative for various values of u_2 will produce solutions which are Pareto optimal for each alternative. After the set of feasible optimal solutions has been generated, a digital computer subroutine can be used to find the non-dominated solution set (NDSS). We will call the NDSS the set Y as shown in Figure 6a. In the policy selection step, the preferences of the DM with respect to the attributes, are used to construct a scalar social choice function (SCF). This SCF ($\hat{Y} = f(u_1, u_2)$ is used to select the best solution from among the NDSS. Assuming that difference independence (in addition to preferential independence) among the attributes holds (Dyer and Sarin, 1979), the form of the SCF is

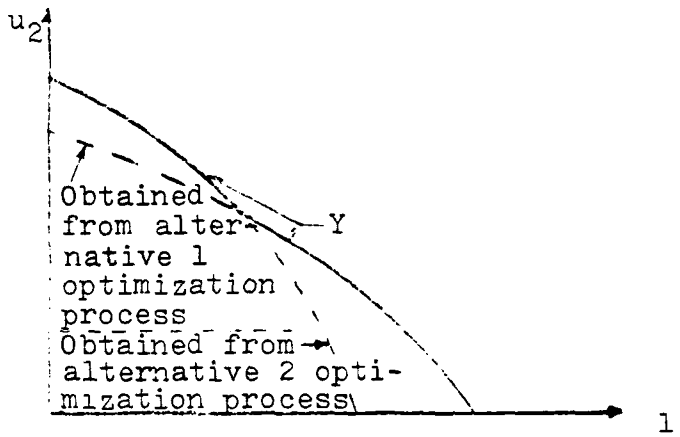


Figure 6a. Non-Dominated Solution Set

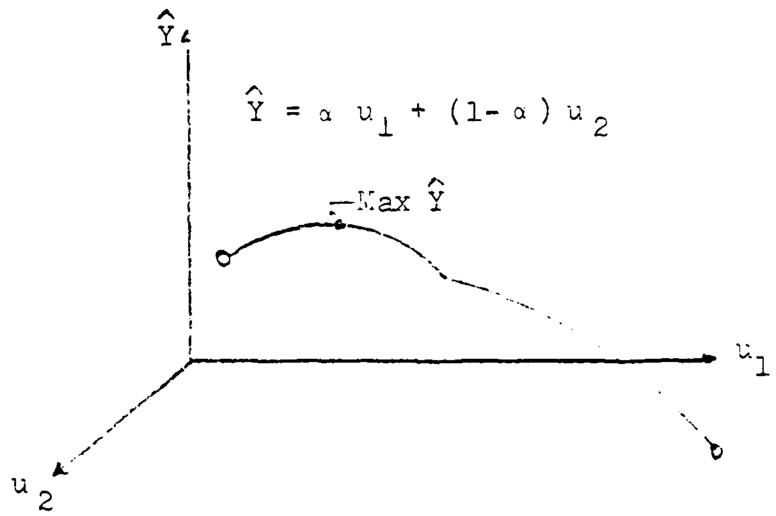


Figure 6b. Evaluation Of Non-Dominated Solution Set With A Social Choice Function

$$\hat{Y} = \alpha \cdot u_1 + (1 - \alpha) \cdot u_2 \qquad (45)$$

After the scaling constant, α, has been assessed (Iyer and Sarin, 1979), the specific solutions in Y (NDSS) are ranked by $\hat{Y}$ and the best solution is identified. This algorithm will score each member of the NDSS and produce a set of $\hat{Y}$ for the NDSS. The set of $\hat{Y}$ is illustrated on Figure 6b. When maximum $\hat{Y} = \hat{Y}^*$ is found, this solution can be traced back to a specific action A_1 or A_2 (depending on whether $\hat{Y}^*$ came from the A_1 or A_2 optimization process). This action (A_1 or A_2) can then be related to the optimum levels of the decision variables $\hat{C}_a$, $\hat{C}_b$, $\hat{L}_a$, and $\hat{L}_b$. The sought-after solution includes the identification of the optimum distribution of commodities $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_1$, and $\hat{b}_2$ which would result from the optimal policy. Iteration of the MOCT procedure along with a comprehensive sensitivity analysis gives the DM an idea of the robustness of the optimal policy. An action plan for implementation of the optimal policy would normally follow.

A numerical realization of the previous example will be presented before extending the example to the case of multiple DMs. Assume that a pre-analysis phase has established that the form of the objective functions for the two consumers is

$$u_1 = a_1 + b_1 \qquad (46)$$

$$u_2 = (a_2 b_2)^{.5} \qquad (47)$$

and two alternative production systems are defined as follows:

Action A_1 (use production system 1)

$$a = f_1(C_a, L_a) = (C_a L_a)^{.5} \qquad (48)$$

$$b = f_2(C_b, L_b) = (C_b L_b)^{.5} \qquad (49)$$

Action A_2 (use production system 2)

$$a = f_3(C_a, L_a) = C_a - 1. \qquad (50)$$

$$b = f_4(C_b, L_b) = (L_b)^{.5} - 2. \qquad (51)$$

In this simplified example, we assume there is no risk or uncertainty. In sequence, the equations from each alternative production system along with the objective functions are substituted into equations 36, 37, 40,

and 41 and the optimization process is carried out. Now each value function is optimized for both alternative systems. The solutions for the proposed alternative systems 1 and 2 and the resulting NDSS are shown on Figure 7. Assume that the attributes are preference independent, and that the SCF has been determined to be

$$\hat{Y} = v u_1 u_2 \tag{52}$$

where v is a constant determining a family of isopreference curves. This SCF is tangent to the NDSS at v = 0.08 at point T in Figure 7. Point T is then the optimum policy of implementing production system 1(A_1) with 5.0 units of capital ($\hat{C}_a$) and 5.0 units of labor ($\hat{L}_a$) going into production of clothing and 5.0 units of capital ($\hat{C}_b$) and 5.0 units of labor ($\hat{L}_b$) going into production of food. This policy will produce the allocation of 2.5 units of clothes (A_1) and 2.5 units of food ($\hat{b}_1$) to consumer 1 ($\hat{u}_1 = 5$), and 2.5 units of clothes ($\hat{a}_2$) and 2.5 units of food ($\hat{b}_2$) to consumer 2 ($\hat{u}_2 = 2.5$). Pending a sensitivity analysis, the optimal policy has been identified for the case of a single DM.

The situation of multiple DM occurs frequently especially in the public sector. To extend this MOOT example to the case of multiple DMs, two DMs will be assumed to each have different attribute preferences. The objectives of both DMs are presumed identical, and the set of system equations are presumed valid. The optimization and subsequent production of the NDSS are carried out as before. The difficulty now arises as to how to combine the individual DM's preferences over the attributes into a SCF. Authors have approached this problem of inter-attribute comparison in a variety of ways (Bankers and Gupta, 1978; Kirkwood, 1972; Eliashberg, 1978; Nakayama, 1979; Sage, 1977) because of the revelations of Arrow's impossibility theorem (Arrow, 1963). The most popular method is a linear weighting of the preferences. Since there are two DMs, a majority voting scheme can be used to establish the consensus weights. Assume that the first DM has the SCF of equation 52 and that the second DM has the SCF of equation 45 with $\alpha = .3$. We assume that the DM's preferences are each weighted equally (i.e., fifty percent for each). The resulting combined SCF is

$$Y' = .5(.08 u_1 u_2) + .5(.3 u_1 + .7 u_2) \tag{53}$$

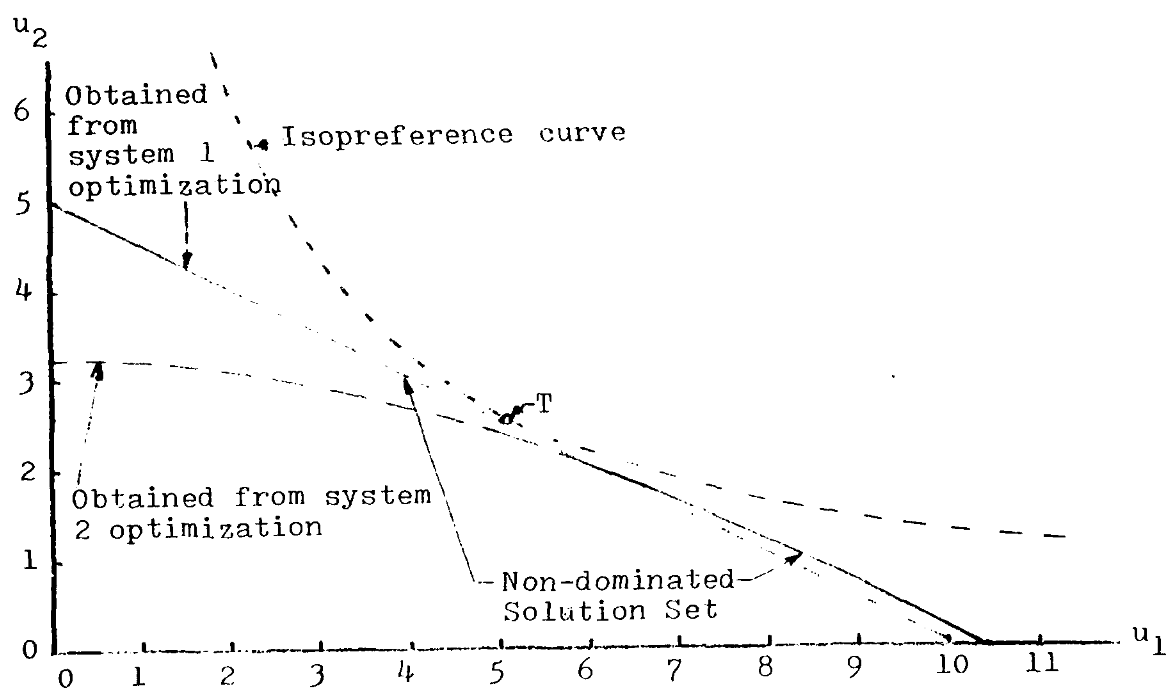


Figure 7. Results Of Welfare Optimization

Now the NDSS shown in Figure 7 is ranked with respect to this scalar SCF to produce the maximum score of $Y' = 2.13$ for alternative A_1. Now A_1 is still implemented with activities of $\hat{C}_a = 5.0$, $\hat{C}_b = 5.0$, $\hat{L}_a = 5.0$ and $\hat{L}_b = 5.0$ but the resulting distribution changes to $\hat{a}_1 = 2.19$, $\hat{a}_2 = 2.81$, $\hat{b}_1 = 2.19$, and $\hat{b}_2 = 2.81$ ($\hat{u}_1 = 4.38$, $\hat{u}_2 = 2.81$). This change in distribution levels was caused by DM #2 increased preference for the utility of consumer 2.

A sensitivity analysis would give feedback as to the validity of this consensus opinion before implementation is actually planned. In this sensitivity analysis we would include an investigation of the impact on optimal policy identification that the variation in scaling parameter values cause.

5. Summary

In this chapter, multiple objective optimization theory (MOOT) was presented as a technique which may be employed to resolve a decision situation. MOOT was first presented at the theoretical level for four cases of a decision situation (deterministic/static, probabilistic/static, deterministic/dynamic, and probabilistic/dynamic). It was noted that while Pareto optimality has the advantage of not requiring interpersonal comparison of preference in a group of DMs, it has a disadvantage in that it is not a complete choice rule (allowing identification of a single optimal policy) since there may be many Pareto optimal alternatives. Then a process algorithm for MOOT was presented at the practical application level in the systems engineering format. At the practice level, it was shown that the MOOT process requires a MAUT technique to accomplish the decision making step. An example is presented of MOOT at the application level using a welfare economics setting. The example illustrates the formation of a NDSS and subsequent selection of the optimal policy from this incomplete ordering.

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CHAPTER 3

MULTIPLE ATTRIBUTE UTILITY APPROACH TO DECISION SITUATIONS

1. Introduction

A multicriterion decision theory (MCDT) approach of interest to the practitioners and theoreticians seeking to resolve decision situations represented by complex alternative acts with many attributes is multiple attribute utility theory (MAUT). This effort presents MAUT at both the theoretical and application level. The MAUT process is applied to a welfare economics decision situation to illustrate the MAUT process.

2. Description Of MAUT At The Theoretical Level

MAUT is basically a decision making theory which requires the analyst to elicit preference information concerning the attributes of proposed alternative policies from the decision maker (DM). Utilizing the DM's preferences, the analyst forms a scalar choice function. Each alternative has associated with it a set of events and outcomes. The event outcomes may occur with either certainty or uncertainty. The scalar choice function is used in conjunction with the corresponding outcomes of each alternative to score and subsequently rank alternative policies for the decision making step.

2.1 MAUT - Certain Outcomes/Time Invariant Case

This MAUT problem can be represented mathematically as

$$\text{maximize } u[h(x,a)] = q(h_1,h_2,,,h_n) \tag{1}$$

$$\text{subject to: } f(x,a) = 0 \tag{2}$$

$$g(x,a) \leq 0 \tag{3}$$

where a is the vector of decision variables, x is a vector of outcome states, u is a scalar value function of the DM (cardinal, or ordinal utility/value function) and q is a function of the attributes h_i (e.g., an aggregation of attributes or of the utility of the attributes) f and g include the linking of policies, events, and outcomes, and other

constraints. Each policy is evaluated and subsequently ranked (this amounts to an exhaustive evaluation over all alternative policies) according to the scalar social choice function (SCF) scoring form, u. This SCF is an amalgamated utility function which is obtained by eliciting the DM's preferences over the attributes (weighing the attributes relative to each other).

The resulting conditions for optimization (identification of the optimal policy) are:

$$f(\hat{x},\hat{a}) = 0 \tag{4}$$

$$g(\hat{x},\hat{a}) \leq 0 \tag{5}$$

$$\frac{\partial u}{\partial h_i}\frac{\partial h_i}{\partial \hat{a}} - \frac{\partial g(\hat{x},\hat{a})}{\partial \hat{a}}\theta - \frac{\partial f(\hat{x},\hat{a})}{\partial \hat{a}}\gamma = 0 \tag{6}$$

$$\theta^T (g(\hat{x},\hat{a})) = 0 \tag{7}$$

$$\gamma^T (f(\hat{x},\hat{a})) = 0 \tag{8}$$

$$\theta,\gamma \geq 0 \tag{9}$$

where $\hat{a}$ is the optimal policy and $\hat{x}$ is the set of optimal outcome states, γ is a Lagrange multiplier vector for the $f(x,a) = 0$ set of constraints and θ is the Lagrange multiplier vector for the $g(x,a) \leq 0$ set of constraints. In order to use the above formulation, a decision situation must have all outcomes occurring at the same time, and each outcome associated with each alternative known with certainty. While a linear SCF is often assumed in the case with certainty of event outcomes, other forms are also possible such as multiplicative or multilinear. Well documented assessment procedures can be used to establish the form of the scalar SCF, to form constituent worth curves, and to evaluate the scaling constants of the SCF (Keeney and Raiffa, 1976; Dyer and Sarin, 1979; Miller, 1967; Edwards, 1977).

2.2 MAUT - Uncertain Outcomes/Time Invariant Case

MAUT is widely used in situations which involve event outcome uncertainty. These events have associated with them outcome state probabilities formed from either empirical data, or subjective data obtained

from the DM in an elicitation process. We now wish to find the alternative which maximizes the expected utility of the DM. For the discrete cases, this is given by

$$\text{maximize } E(u) = \text{maximize } \sum_j p_j(a_i)u(z_{ij}(a_i)) \qquad (10)$$

over each alternative a_i where E is the expectation operator, p_j is the probability of the j^{th} event outcome, z_{ij}. The scalar SCF, u, is a utility function which incorporates the DM attitude toward risk. This function can be formed by combining the attributes into a single attribute and then transforming it to a risky utility function (Boyd, 1970), or by forming constituent utility functions for the attributes and then aggregating these into a scalar function. Many authors including Keeney and Raiffa (1976) describe assessment procedures for this latter method which enable one to discern the form of u and to identify scaling parameters based on the relationships among the attributes.

The alternative policies, events, outcomes and uncertainty in a MAUT formulation can usually be displayed in a decision tree format such as the simplified single stage tree shown in Figure 1. In a special case of the single stage decision problem, which includes probabilistic elements, if the event outcomes are the same for each alternative policy, then it may be possible to identify the optimal policy without determining the utility for the outcomes (assuming the outcomes can be ordered on a monotone scale) by employing the concept of stochastic domination (Bunn, 1978).

2.3 MAUT - Time Varying Case

In Sections 2.1 and 2.2, time was included in the concept of an outcome state which contained a summary of many circumstances of which terminal time was at least implicitly considered. For example, an event outcome may be to realize a favorable reward from a business endeavor. This outcome does not specify a specific time dependency, instead it is meant that at a terminal time in the future, the favorable business reward will be realized. Hence, utility assessment is conducted without explicitly bringing in a time dependency. A difficulty in

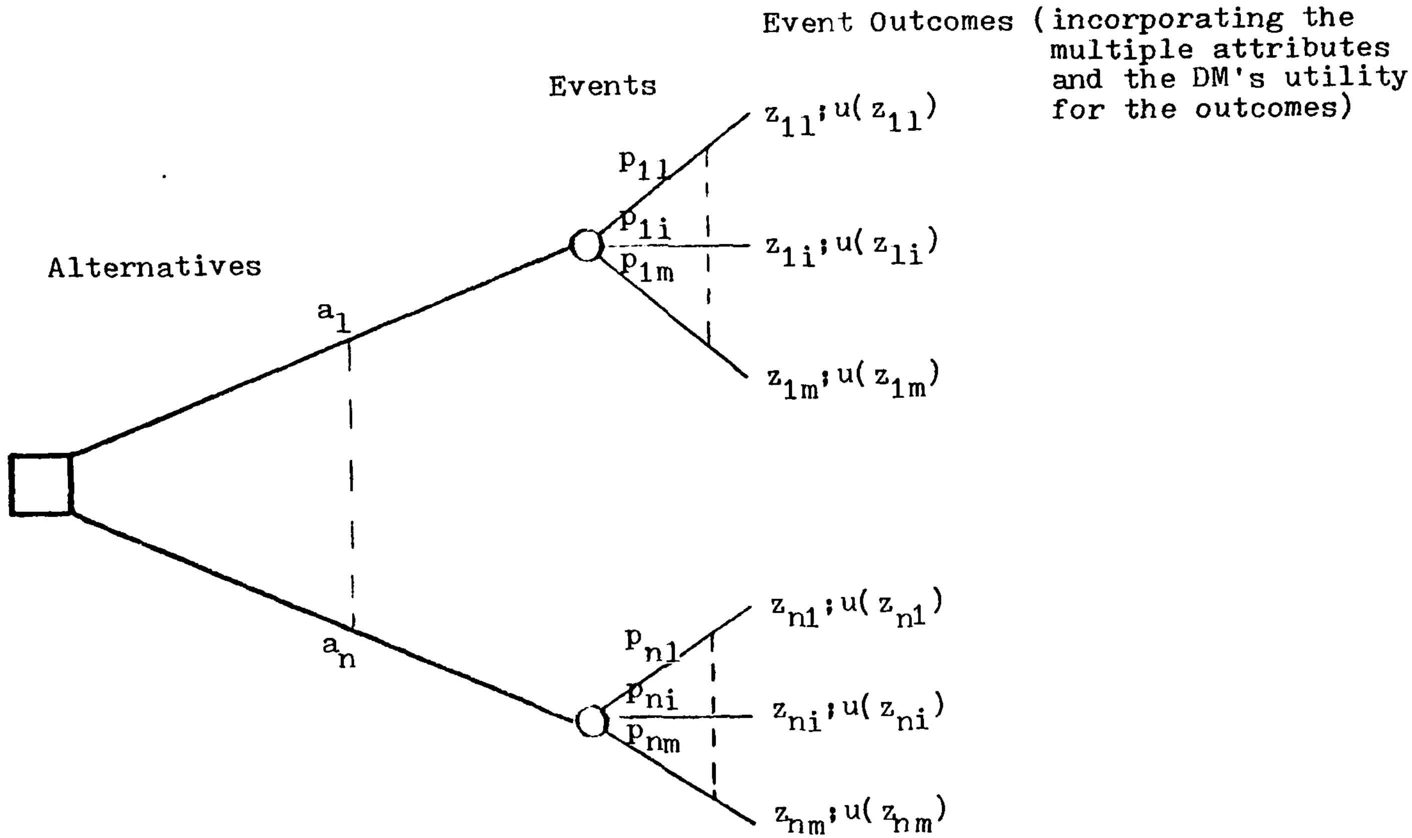

Figure 1. Single Stage Decision Tree Representation Of The Decision Situation

incorporating time into MAUT concerns the assessment of utility under time varying conditions. The concept of assessing the DM's utility as a function of his or her value for an attribute and on the passage of time makes for a very difficult assessment. Other problems can occur in the evaluation of E [u(x)] if x evolves from a time dependent process. For example, in the time dependent case, equation (2) becomes

$$dx = f(x,a,t)\ dt + Q\ (t)\ dw \tag{11}$$

where dw is the realization of a random process (for example a Weiner process). While x(t) and its moments can be determined, there are problems evaluating

$$E[u(x)] = \int u(x)\ p(x)\ dx \tag{12}$$

where x evolves from (11) unless u(x) is linear or quadratic.

If all outcomes occur at specific points in time, it is reasonable to discount the utility or expected utility over time as a way of bringing in a sense of dynamics. This discount factor incorporates the concept that immediate effects are of more value than future events. For the risky case of MAUT, the SCF becomes

$$E(u) = \left(\sum_{k=1}^{L} [\alpha^{k-1} \sum_j p_j(a_i) u(z_{ij}(a_i))] \right) \tag{13}$$

where α is the discount factor and L the number of periods.

A tractable technique which is occasionally used is the practice of costing out the attributes and combining their value into a pecuniary sum which can be used in a discounted time stream. Here time is included explicitly along with a discount rate. The social choice function now can take the form of

$$C = \sum_{i=1}^{L} \alpha^{i-1}\ C_i \tag{14}$$

where C is the present net worth of the income stream, C_i is the income for the i period, i=1,,,,,,L, which is a result of costing out the attributes for the outcome of that period. The alternative with the highest value of present net worth is the preferred alternative.

3. Multiple Attribute Utility Process

Multiple Attribute Utility Theory (MAUT) applied at the practice level describes techniques which determine the DM's utility and select the optimal course of action as a solution to a decision situation. Multiple Attribute Utility Theory is currently the object of considerable research and application, and it is likely that there are differences of opinion among practitioners and researchers on the scope and nature of this approach as used in practice. While it is basically an algorithm for decision making, some practitioners of multiple attribute utility theory include all systems engineering steps within the algorithm, as does this description.

3.1 Description Of The MAUT Process With Uncertain Outcomes

A MAUT process involving uncertain outcomes is now described. The use of MAUT is predicated on the following assumptions:

1. Feasible sets of alternative actions and the events and event outcomes resulting from the alternative acts can be identified.

2. The cause-effect relationships between alternative actions and the outcomes can be determined so that the decision situation model mimics reality adequately.

3. A set of attributes must be defined which measure the attainment of the objectives of the decision situation. The DM will provide preference information over this set of attributes (which also describe the characteristics of the outcomes) in a form which will allow his or her felicity for each outcome to be expressed in a scalar utility score.

4. Probabilistic information concerning the outcomes is available from empirical data, or it can be obtained with a subjective assessment from appropriate personnel.

Output Results of the MAUT Process

Successful application of the MAUT process should result in a realistic model of the decision situation (often in graphical form) in terms of action alternatives and outcomes. The output from a MAUT process should include a quantification of the DM's preferences with

respect to the event outputs, an identification of trade-offs present in the decision situation, an identification of the optimal decision policy, a determination of the costs and benefits of the optimum strategy, and an indication of the robustness of the solution.

MAUT Process Algorithm

The application of MAUT involves many steps which should be accomplished in an iterative manner.

a. A pre-analysis phase defines the scope and boundary, and identifies the elements of the decision situation. These elements include the needs, constraints, alterables, objectives or goals, and attributes (e.g., let attribute h_1 represent cost, and h_2 represent performance). A set of decisions or alternative policies, (e.g., let a_1 represent a decision to buy commodity 1 and a_2 the decision to purchase commodity 2) which can resolve the decision situation, is identified.

b. A systems analysis/modelling phase accomplishes the structuring necessary to define a finite set of possible and reasonable events and associated event outcomes for the decision situation. These event outcomes are described in terms of the attributes (e.g., let a set of outcomes be z_i where $z_i = (h_1^i, h_2^i)$) so that their merit can be related back to achieving the objectives of the decision situation. The modelling continues by identification of the relationships between the decisions and the event outcomes. The alternative policies may require refinement and tuning which is efficiently accomplished using a mathematical optimization technique. The original alternative policies or a selected number of the refined forms of the alternative policies become the actual decision options which are linked to the outcome states through established relationships. Since these relationships are governed by risk, the causal relationship of the decision, event, and subsequent event outcome can be represented efficiently in a decision tree. Since outcome uncertainty is present, encoding of subjective probability from experts or the DM is a viable approach to identifying and quantifying the uncertainty.

Since the outcomes are combinations of events and decisions, the DM's value (e.g., $v\ (z_{ij}(a_i))$ where v is the value of outcome z_{ij} caused

by decision a_i) of an outcome (function of attributes) can be used to comparatively score the alternative action that caused the outcome. Once the decision situation is modelled in MAUT, a preference assessment effort is conducted. The purpose of the preference assessment is to elicit information required to construct the DM's preference function for the various attributes which describe the possible outcomes (Keeney and Raiffa, 1976). Since we are assuming uncertainty over outcomes, this preference function is in the form of a cardinal utility function which incorporates the DM's attitude toward risk. This preference function is in effect a scalar social choice function (SCF). The SCF can be formed in two ways (Keeney and Raiffa, 1976):

1. Assigning a scalar utility score to a scalar value function: This function has incorporated the DM's value of the outcome in an acceptable form. This value function is generally a scalar, $v = f(v_1,,v_i,,v_n)$ where v_i is a value function of the ith attribute. An assessment of a scalar utility function is all that is then required to transform the value function score to a utility function score (e.g., the utility SCF is $u = f(v(z_j(z_i)))$. This wholistic approach is suggested by many such as Boyd (1970). This approach may be aided by deterministic aids such as SMART (Edwards, 1977) or policy capture (Hammand, et. al., 1977).

2. Establishing relationships between the attributes directly with respect to preference and utility independence. These relationships dictate the functional form of the SCF of utility. Once this form is established, the utilities for the attributes are assessed and scaling constants (weights) are evaluated to define the SCF (e.g., the utility SCF is $u = f[u_1(x_1),u_2(x_2)]$. Now the outcomes, in terms of their attribute levels, are transformed to a scalar utility score. When a group has responsibility for the decision, this preference elicitation process becomes more complex. Various authors have addressed the problem of group consensus to form a SCF (Arrow, 1963; Keeney, 1976a; Harsanyi, 1955; Keeney and Kirkwood, 1975; and Nakayama, et al., 1979).

After all the outcome values or characteristics have been replaced with the DM's utility for those outcomes, the decision model is ready

to be optimized (e.g., z_i is replaced by $u(z_i)$.

c. Ranking of the alternative actions/policy selection basically involves maximizing the DM's utility over the set of decisions. This subjective utility aggregation involves averaging out utility scores for each decision, and folding back the decision model if it is a staged or sequential model involving probabilistic concepts (e.g., max $E(u) = \max_a \sum_z [p_j(a_i) \cdot u(z_{ij}(a_i))]$. Since the criterion is the expected value of a scalar SCF, an ordered ranking of alternative actions is an available result.

d. Sensitivity Analysis is conducted to investigate modelling errors. This exercise can contain an analysis of the value of gathering more information to refine the accuracy of likelihood estimates.

e. An algorithm for the MAUT process is shown in the DELTA chart of Figure 2. The policy selected in the decision making step is used as the input to the planning for the action step. The specific activities in the modelling, ranking of alternatives/decision making steps are shown in Figure 3.

MAUT is appropriate for identifying the optimal course of action using utility theory techniques when the size or complexity of the decision situation makes intuitive solutions extremely difficult to obtain. MAUT is adept at handling situations with static or sequential settings concerned with risk, uncertainty, or deterministic data, and the DM's attitude toward any risk. Many MAUT techniques are particularly adept at handling behavioral motivated criteria because of their ability to model subjective quantities as well as incorporate qualitative information. The MAUT approach is well suited for application to many public and private decision areas such as evaluating risky ventures, and identification of preferred courses of action.

Input Requirements for the MAUT Process

Information in usable form needs to be available for interrelating the salient elements of the decision situation. This information includes: a set of attributes which measure the attainment of the objectives and goals; a set of feasible decisions as proposed solutions

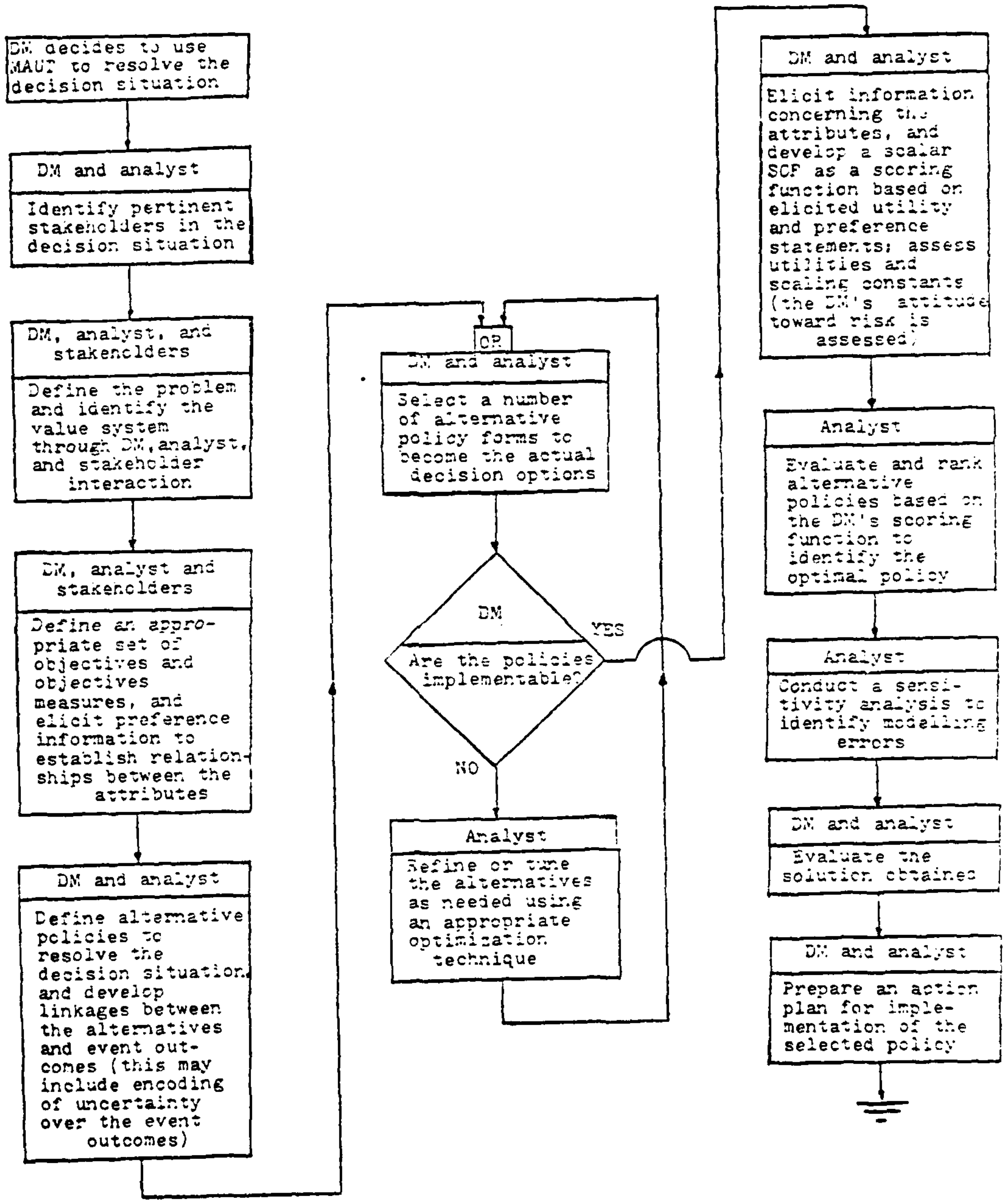

Figure 2. DELTA Chart For MAUT Process With Uncertain Outcomes

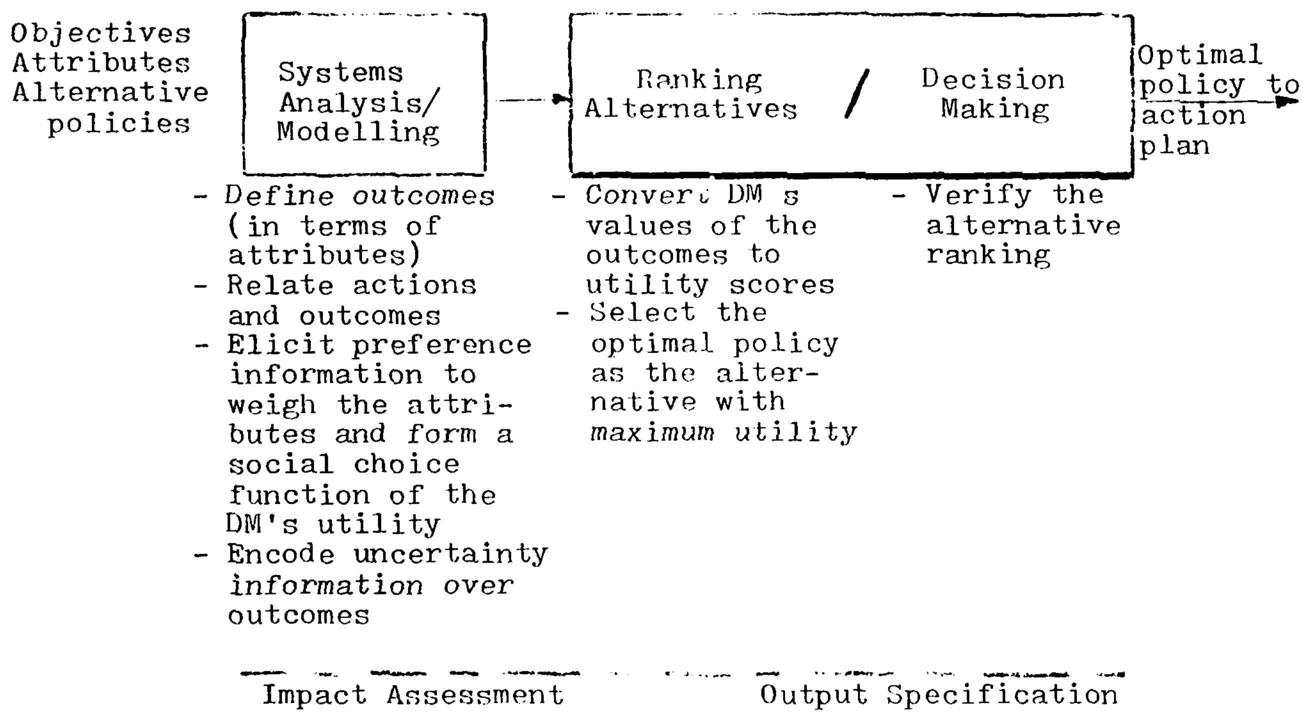

Figure 3 . MAUT Formulation

to the decision situation; set of outcomes (described in terms of the attributes) as possible results of the decisions; the relationships between the alternative actions and the outputs; valuation of the outcomes using the preferences of the DM over the attributes and outcomes, and possibly the DM's attitude toward risk.

Essential personnel required for the MAUT process are the analyst(s) experienced with MAUT techniques, and the DM(s) who are responsible for a solution to the decision situation. Generally other stakeholders and experts are also involved in the MAUT process in order to aid in the modelling effort.

Recommended facilities for use in a MAUT process are graphical display devices (CRT, CRT projector, view-graph projector, chalkboard, charts, etc.) and a digital computer to aid in interrelating elements, encoding probabilities, eliciting preferences, ranking the alternatives and displaying results. These aids process and present information visually which will often facilitate the interaction between the analyst and stakeholders.

The integrity of the MAUT process and identified optimal policy can be checked by the following set of questions: Does the decision model of MAUT adequately mimic the reality of the decision situation so as to give the DM confidence in the indicated results? Are the results consistent with the objectives and goals of the DM and organization responsible for the decision situation? Does the DM understand the model sufficiently to give confidence in the results? Have trade-offs required in the resolution of the decision situation been identified? Are the results of MAUT in a form which can be implemented by the DM? Could intuitive judgment have been used to produce a solution of comparable quality?

Additional Information

There are many computer programs available to aid the analyst in a MAUT application. The greatest number of programs to aid in preference elicitation and subsequent optimization of the decision model have been associated with the technique of decision analysis (e.g., CTREE, MUFCAP, PEP, MANECON) as discussed by Schlaifer (1971), Keeney and Sicherman

(1975). There are also programs designed to aid in utility assessment as discussed by Freedy et al., (1974) and Ulvila (1975).

MAUT has been applied to applications such as the selection of development strategies for airport facilities for Mexico City (Keeney and Raiffa, 1976), the identification of policy for land-use regulation in California (Edwards, 1977), the evaluation of emergency communications systems (Yorke, Gianniny, and Sage, 1978), the selection of optimal policy for a metal broker to follow in purchasing iron ore (Brown, Kahr, and Peterson, 1974), the selection of an optimal policy for location of a house for the family to reside in (Nakayama, et al., 1979), and the selection of policy for water resource planning (Keeney and Wood, 1977).

3.2 Variations In The MAUT Process With Certainty Of Outcomes

The major difference in the probabilistic and deterministic MAUT processes concerns the uncertainty of outcomes. The deterministic case assumes that the outcomes from the different alternative policies are known with certainty. This makes it unnecessary to incorporate the DM's attitude toward risk or to assess subjective probabilities for the events. Value or worth scores for the attributes can be assessed, and the form of the SCF can be found from the relationship of the attributes. Often, a linear form of SCF is used (the attributes are assumed independent) in the case of certain outcomes (Miller, 1967; Edwards, 1977). There is now no need for an expectation operator as each alternative is scored by the deterministic SCF. This scoring allows a ranking of alternatives and subsequent decision making step.

4. An Example of MAUT

To illustrate the MAUT process, and as a way of producing a basis for comparison of the conditions required for optimization in the MAUT process and MOOT process, we will consider the same example from the area of welfare economics as presented in the previous chapter. MAUT is a valuable technique because it can be utilized to resolve the indeterminancy which arises if Pareto optimality is a requirement for

welfare optimization. We assume that society is concerned with equity (all members receiving the same level of satisfaction with the consequences) as well as efficiency (Pareto optimality). We further assume that a benevalent DM will combine the utilities of its members into a scalar function which reflects the aggregate utility of society. One can generally find a scalar objective function which has been maximized for any particular optimal resource allocation. In welfare economics, this scalar social choice function (SCF) or social welfare function (SWF) representing societal utility is specified and a resource allocation determined to maximize this social welfare function. The SWF is then a convenient form to use to maximize group utility while preserving equity considerations (Keeney and Kirkwood, 1975; Bodily, 1976).

Consider again the following formulation of society's economic problem. For concreteness, we shall assume a simple closed economy with two consumers. We will assume that the economy is endowed with two factors, capital C and labor L. There are two outputs from production, a (clothing) and b (food). There are fixed endowments of labor and capital $\bar{C}$ and $\bar{L}$ described by

$$\bar{C} = C_a + C_b \tag{15}$$

$$\bar{L} = L_a + L_b \tag{16}$$

where $\bar{C}$ and $\bar{L}$ are the maximum levels of factor supplies, C_a and C_b are the amount of capital allocated for producing a and b respectively, and L_a and L_b indicate the amount of labor which is allocated for producing a and b respectively. The production functions which determine the amount of clothing and food produced are given by

$$a = f_i\ (C_a,\ L_a) \tag{17}$$

$$b = f_j\ (C_b\ ,\ L_b) \tag{18}$$

Two alternative actions (A_1, A_2), which are each composed of different production functions, have been defined. Each alternative action is composed of the four policy variables C_a, C_b, L_a, and L_b. In this example, not only will the optimum alternative which maximizes the utility of the two consumers be identified, but also the best levels of

the policy variables for the optimum will be found. We assume no risk or uncertainty in this simplified example.

We assume that all agricultural and clothing production will be distributed between the two consumers as needed to maximize their happiness. Thus we have

$$a = a_1 + a_2 \tag{19}$$

$$b = b_1 + b_2 \tag{20}$$

where a_1 and a_2 are the amount of clothes allocated to consumers 1 and 2 respectively, and b_1 and b_2 are the amount of food allocated to consumers 1 and 2 respectively. The objective of each consumer is to maximize his or her utility.

We assume that the utility of each consumer depends directly on the quantity of products consumed. The utility functions for the consumers are assumed to be the isotone functions

$$u_1 = g_1 \ (a_1, b_1) \tag{21}$$

$$u_2 = g_2 \ (a_2, b_2) \tag{22}$$

Since we are concerned with egalatarian as well as efficiency considerations, let the SWF or SCF be

$$U = h \ (u_1, u_2) \tag{23}$$

If we now maximize with respect to this SCF, we have in effect a MAUT formulation as

$$\text{Max} \quad U \tag{24}$$

$$\text{subject to:} \quad u_1 = g_1 \ (a_1, b_1) \tag{25}$$

$$u_2 = g_2 \ (a_2, b_2) \tag{26}$$

$$\overline{C} = C_a + C_b \tag{27}$$

$$\overline{L} = L_a + L_b \tag{28}$$

$$a = f_i \ (C_a, L_a) = a_1 + a_2 \tag{29}$$

$$b = f_j \ (C_b, L_b) = b_1 + b_2 \tag{30}$$

Now adjoining the constraints to U with Lagrange multipliers, extremizing to obtain the necessary conditions, and then manipulating these

conditions to eliminate the Lagrange multipliers yields the three previous efficiency equations which resulted from the optimization of the component utility functions in the MOOT example (equations 42, 43, and 44; Chapter 2) plus the social equity equation

$$\frac{\dfrac{\partial u_1}{\partial a_1}}{\dfrac{\partial u_2}{\partial a_2}} = \frac{\dfrac{\partial U}{\partial u_2}}{\dfrac{\partial U}{\partial u_1}} \tag{31}$$

where $\frac{\partial U}{\partial u_1}$ and $\frac{\partial U}{\partial u_2}$ indicate the marginal effects on U by the utility of consumer 1 and 2 respectively. This last equation in effect picks a point on the NDSS of MOOT formulation. This point is the optimal policy that is the most desirable with respect to the SWF. From this example, it is shown that the necessary conditions for optimality of social welfare include all the necessary conditions for Pareto optimality plus an extra condition pertaining to egalatarian considerations. The analyst optimizes the SWF for each alternative production system to produce values for allocation of resources and commodities, and a scalar score reflecting the overall utility due to each alternative production system. These scores are used by the DM as the basis for identifying the optimal policy. Following a sensitivity analysis and iteration, the DM selects the optimum policy and proceeds to plan for implementation.

The form and parameter values of the SWF, U, are determined from information gathered through preference elicitation (Keeney and Raiffa, 1976; Dyer and Sarin, 1979; Edwards, 1977).

A numerical realization of the previous example will be presented before extending to the case of multiple DMs. Assume that a pre-analysis phase has established that the form of the objective functions for the two consumers is

$$u_1 = a_1 + b_1 \tag{32}$$

$$u_2 = (a_2\ b_2)^{.5} \tag{33}$$

and two alternative production systems are defined as follows:

Action A_1 (use production system 1)

$$a = f_1 (C_a, L_a) = (C_a L_a)^{.5} \tag{34}$$

$$b = f_2 (C_b, L_b) = (C_b L_b)^{.5} \tag{35}$$

Action A_2 (use production system 2)

$$a = f_3 (C_a, L_a) = C_a - 1. \tag{36}$$

$$b = f_4 (C_b, L_b) = (L_b)^{.5} - 2. \tag{37}$$

In this simplified example, we assume there is no risk or uncertainty. We assume preferential and utility independence such that the form of the SCF is

$$U = k_1 u_1 + k_2 u_2 \tag{38}$$

where k_1 and k_2 are scaling constants which have been assessed to equal .3 and .7 respectively. In sequence, the equations from each alternative production system along with the objective functions and SCF are substituted into equations 25, 26, 29, and 30 and the optimization process is carried out. The resulting score for alternative 1 is 3.50 and the score for alternative 2 is 3.05. Therefore the optimum policy is to implement production system 1 (A_1) with 5.0 units of capital ($\hat{C}_a$) and 5.0 units of labor ($\hat{L}_a$) going into production of clothing and 5.0 units of capital ($\hat{C}_b$) and 5.0 units of labor ($\hat{L}_b$) going into production of food. This policy will produce the allocation of 0.0 units of clothes ($\hat{a}_1$) and 0.0 units of food ($\hat{b}_1$) to consumer 1 ($\hat{u}_1 = 0$) and 5.0 units of clothes ($\hat{a}_2$) and 5.0 units of food ($\hat{b}_2$) to consumer 2 ($\hat{u}_2 = 5$).

A sensitivity analysis, which would include an examination of the variation in the optimal policy due to changes in the form and scaling constants of the SCF, would provide a basis for validation of the resulting policy for the DM.

To extend this example to multiple DMs, it is assumed that there are now two DMs who will decide on which production function to utilize for society. Each of these DMs is assumed to have their own preference for the utility of the consumers. The problem of how to amalgamate

the two DM's preferences into a single SCF faces the analyst. The difficulty in interpersonal comparison of utilities has been addressed by Keeney (1976), Kirkwood (1974), and Arrow (1955), Nakayama, et al., (1979), Sage (1977) and others. Becuase there are two DM's, a majority voting scheme can be used to combine the individual additive utility functions. In a decision situation with more than two DMs, the works of Keeney (1976), Kirkwood (1979), and Eliashberg (1978) and others can be of help. Since an additive combined SCF is assumed, each DM's SCF is of the form

$$U' = \alpha_i u_1 + (1 - \alpha_i) u_2 . \tag{39}$$

The table below shows each DM's values:

DM#	α_i	$1 - \alpha_i$
1	.3	.7
2	.9	.1

The resulting SCF U' is formed by averaging the individual scores (2 voters with equally weighted opinions)

$$U' = (.5\,\alpha_1 + .5\,\alpha_2)u_1 + (.5(1 - \alpha_1) + .5(1 - \alpha_2))u_2 \tag{40}$$

The resulting SCF is

$$U' = .6\,u_1 + .4\,u_2 \tag{41}$$

Using this SCF, the alternative scores become 6.0 and 6.1 for production system 1 and 2 respectively. These scores indicate that alternative 2 should now be implemented with activities of $\hat{C}_a = 10.0$, $\hat{C}_b = 0.0$, $\hat{L}_a = 0.0$, and $\hat{L}_b = 10.0$ thereby causing a distribution of commodities of $\hat{a}_1 = 9.0$, $\hat{a}_2 = 0.0$, $\hat{b}_1 = 1.16$, and $\hat{b}_2 = 0.0$ ($\hat{u}_1 = 10.16$, $\hat{u}_2 = 0$). Pending a sensitivity analysis, the optimal policy has been identified for the case of the two DMs.

5. Summary

This chapter presents multiple attribute utility theory (MAUT) as a multiple criteria approach to resolution of a decision situation. MAUT is presented at two levels (theoretical development and practical

application) as a way of elucidating the approach and to develop a comparison basis with the MOOT presentation of Chapter 2. MAUT is presented at the theoretical level for the decision situation cases involving certain and uncertain outcomes. At the practical application level, the presentation of MAUT in the systems engineering format shows that the MAUT process often needs a MOOT technique to refine the decision options. A welfare economics motivated example is presented that shows the necessary conditions for maximizing social welfare include all the necessary conditions for Pareto optimality plus an extra condition to preserve equity considerations. The SWF criterion utilizes a MAUT approach which resolves the indeterminacy of a Pareto efficiency criterion in the sense that it will select a specific point on the Pareto frontier.

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CHAPTER 4

A COMBINED MOOT/MAUT APPRCACH TO DECISION SITUATIONS

1. Introduction

Our purpose in this research is to motivate and present a more efficient MCDT approach which combined features of both the MOOT and MAUT approaches. In the previous chapters, detailed descriptions of MOOT and MAUT processes are provided as useful frameworks to generate solutions to decision situations, and as a basis for subsequent comparisons of both approaches. This comparison, which shows the presence of certain efficient steps in the approaches and the lack of structural differences, leads to a combined MOOT/MAUT approach. This approach is based on the complementary aspects of both approaches.

Section 2 presents a comparison of MOOT and MAUT with respect to their generic structure, function, purpose and organizational implications. Section 3 describes a combination MOOT/MAUT approach which stresses the complementary nature of MOOT and MAUT. A DELTA chart is presented as an aid in selection of an appropriate MCDT approach to a decision situation. An example of the combined MOOT/MAUT approach is presented to illustrate it in an application.

2. Comparison of MOOT and MAUT Processes

2.1 Structural Comparison

The structural basis of both MOOT and MAUT processes produces a set of common characteristics. Descriptive of the way either approach actually organizes the decision situation elements, these characteristics include:

* a set of objectives (generally non-commensurate and conflicting) which reflect the DM's values
* a set of attributes for measuring attainment of objectives
* a set of alternative actions or decision variables
* an elicitation of preferences concerning the attributes from the DM (required to preserve contextural integrity)

This set of common characteristics leads one to expect the same quantitative solution from both MOOT and MAUT processes when they are applied to the same decision situation. Indeed, (as confirmed in Appendix B), this is the case because the MOOT process eliminates dominated sets of solutions which are not Pareto optimal. Surely none of these sets can be optimal in MAUT where Pareto optimality is a necessary condition. The statement above requires assumptions that the DM is consistent with respect to the preferences elicited in the MOOT and MAUT processes and that both MOOT and MAUT are applied at the same level of practice so the perceptions of the DM are on the same level.

Referring to the process and algorithm descriptions in Chapter 2 and 3, one can conclude that MOOT and MAUT processes (at comparable levels) are both mental constructs to approaching multiple criteria decision situations and that for all intents and purposes, there are no fundamental differences in structure between them. Both approaches must be applied at the same level to insure that the perception of the situation is identical to the DM, otherwise there is a possibility of a structural difference arising due to perception alone.

It is because of the structural similarity of the approaches that a given decision situation can generally be posed using either MOOT or MAUT processes to produce a strategically equivalent optimal policy.

2.2 Function And Purpose Comparison Of MOOT And MAUT

MOOT and MAUT were each developed for different purposes which are indicated by their respective titles. Multiple Objective Optimization Theory is designed to refine or tune a set of alternative policies and indicate impacts using a mathematical optimization technique to eventually form the NDSS. This mathematical optimization requires precise situation process descriptions, but allows the notion of time to be explicitly included in the model. At the practice level, MOOT separates the optimization and decision making steps by requiring a MAUT technique for the policy selection or decision making step. Since a MAUT technique will be used for the decision making step, detail elicitation will be eventually required to establish the form and constituents of the

scalar SCF. Since the formation of the SCF requires preferential independence (PI) of the attributes, this condition needs to be established in all multiple criteria techniques to insure that the decision making step is mathematically based and an optimal policy can be identified from the NDSS. MOOT, therefore, is essentially designed as an impact assessment tool. Alternately, Multiple Attribute Utility Theory is designed to pose the situation attributes in terms of the DM's utility and then maximize this utility over the policy alternatives. At the practice level, MAUT combines the optimization/ranking and decision making steps (discounting any portion of the optimization process required to tune or refine the alternative policies). MAUT is essentially a decision making tool.

A major operational difference which is apparent at the practice level is that the scalar social choice function required for selection of the optimal solution is formed after the vector optimization and generation of the non-dominated solution set in MOOT, and before ranking alternatives in MAUT.

Mathematicians and engineers have generally been involved in the development of MOOT. It is not surprising then that the types of situations which have generally employed MOOT techniques are those where all parts of the problem are quantifiable (because of the mathematical optimization process). The relationships in the problem can include time dependencies, and there are physically motivated functions, such as in resource allocation and profit maximization situations. A prime area of MOOT application is in the determination of optimum policy parameters for hardware subsystem design.

Not only have those with quantitative interests such as mathematicians, economists, and engineers been involved in MAUT, but also psychologists and other behaviorists have made many contributions to this area. This influx of behavioristshas made MAUT more robust in areas of application. The application areas of MAUT overlap those of MOOT and extend to situations where parts of the problem formulation are not quantifiable. Many MAUT techniques facilitate the quantification of the qualitative parts of a decision situation. MAUT is well suited to

situations involving behaviorally motivated criteria. This wide area of application makes MAUT a frequently used decision tool. MAUT techniques are designed to indicate optimal policies when the decisions are in the form of a combination of an alternative action and a set of discrete levels of the decision variable. This is the form of the decisions in many situations with limited feasible options (i.e., to use a current system or acquire a new system).

The concept of independence of objectives and attributes with respect to the DM's preference and utilities is of paramount importance in MAUT at the practice level. This independence property is linked to the general form of the SCF, and must be verified by the preference elicitation process (Keeney and Raiffa, 1976).

2.3 Organizational Implications Of The Application Of MOOT And MAUT At The Practice Level

When either MOOT or MAUT approaches are used, there is a commitment made by the using organization (organization here is used in the context of a group of stakeholders including DM(s) and analyst(s) in either the public or private sectors who are interested in resolution of the decision situation) to dedicate the required resources toward producing a solution. Because of the structural similarity of MOOT and MAUT processes, much of the information required concerning a decision situation by either process is for all practical purposes identical. Likewise the time requirement to carry out MOOT and MAUT processes is comparable because similar functions must be accomplished with the DM's and other stakeholders in both processes. There are organizational implications of the operational differences in MOOT which allow the DM to express alternative value scores prior to optimization, and the criterion weights (for the SCF) following formation of the NDSS. In the case where contact between the analyst and DM is limited to separate short intervals, the MOOT process should be more efficient because the analyst can accomplish the part or all of optimization process to form the NDSS between interviews. In MAUT, both the DM's value scores and criterion weights must be accomplished prior to ranking alternatives

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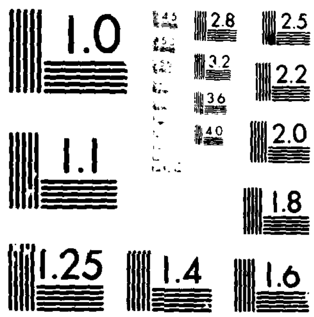

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which may delay somewhat, the identification of the optimal policy in the case of restricted DM-analyst interaction mentioned above. Very often, limited contact with the DM(s) and the other stakeholders necessitates certain assumptions such as linearity and risklessness as discussed by Edwards (1977). While these assumptions limit the types of multiple criteria analysis techniques which can be used, their presence often allows for a solution in a pragmatic manner. In these situations of restricted DM-analyst contact, the analyst is challenged to organize the interview (utilizing techniques such as prior construction of attribute templates for verification by the DM and rapid screening of decision options - Selvidge, 1976) so that the essence of the DM's values can be rapidly elicited.

Before the user proceeds with either a MOOT or MAUT process, the benefits and costs of using a specific technique should be examined. Certain authors suggest benefit limits as a GO-NO GO threshold for commiting resources to a specific technique, or the cost swing (the difference in cost between the least expensive and most expensive alternative solution) as a limit of the maximum which should be spent on analysis for resolving a decision situation (Brown and Ulvila, 1976). A figure for the cost of the application of a MCDT approach can be obtained from estimates of personnel experienced in this area, or from the bids of consulting firms who are engaged in this business.

3. A Combined MOOT/MAUT Approach

3.1 Complementary Aspects Of MOOT And MAUT

The comparisons just presented point out that MOOT and MAUT have each developed as methods primarily directed at two different purposes. MOOT is efficient at optimizing in the multiple criterion case while MAUT is adept at incorporating the DM preference structure into the decision making effort. As demonstrated in Chapters 2 and 3, a pragmatic application of either MOOT or MAUT requires some interaction of both approaches. It is logical to deduce, that the most is gained by using each approach in a complementary fashion in the manner for which each

was developed. In this way a multiple criteria approach described in terms of systems methodology takes on an air of synergism as steps five (optimization/ranking of alternatives) and six (decision making) contribute efficiently to the resolution of the decision situation.

3.2 Algorithm For The MOOT/MAUT Approach

If it appears cost effective to use a MCDT approach to resolve the decision situation, then Figure 1 is an aid which can lead the user to an appropriate technique. The DELTA chart of Figure 1 indicates to the analyst and DM whether the combined MOOT/MAUT approach is appropriate for a specific decision situation. The algorithm for the joint approach is shown in Figure 2 as outlined below. This algorithm resembles the algorithm of the MOOT process at the practice level because of our non-traditional extensive description of the MOOT process. The deterministic/non-time varying case is described in this algorithm because of its generality.

a. A pre-analysis phase is accomplished to generate the input description and specification as discussed in Section 3.1 of Chapters 2 and 3.

b. A systems analysis/modelling phase is directed toward constructing the situation process so that impacts of the various alternative policies, as discussed in Section 3.1 of Chapters 2 and 3, will be produced.

c. Identification of feasible alternatives is the next step prior to optimization.

The generation of a set of alternative actions from which to choose is accomplished in system synthesis. In an effort to cause the DM to think in a comprehensive and thorough manner, the analyst should encourage the DM to consider as many alternatives as possible. Analysts and others on the DM's staff may be of considerable assistance in this regard. Logical steps in identifying the feasible alternatives are to list the possible alternative acts and then eliminate the improper acts. This reduction of the decision space to include the feasible alternatives is usually accomplished to some degree by the DM (perhaps subconsciously)

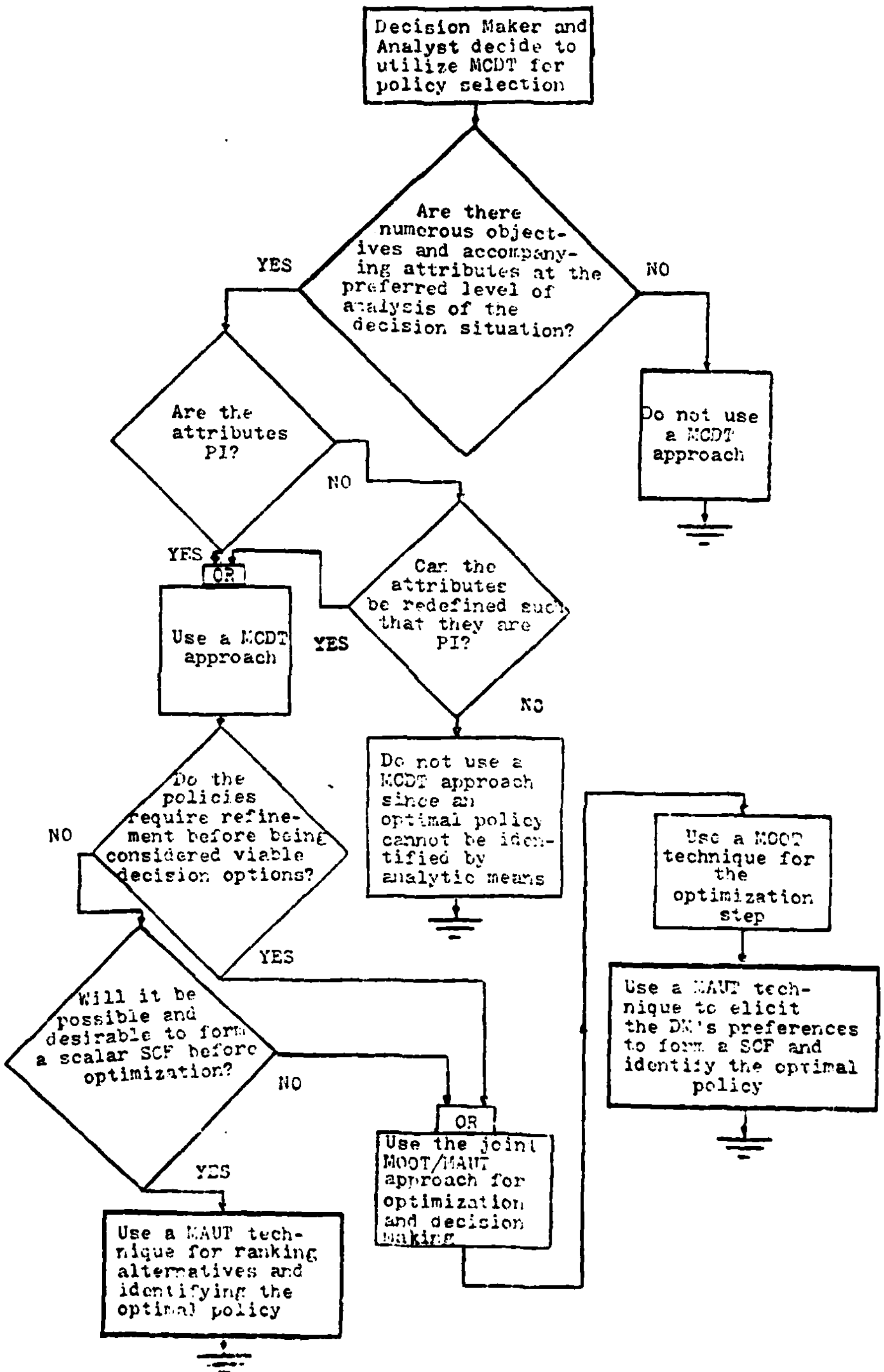

Figure 1. MCDT Approach Selection For A Decision Situation

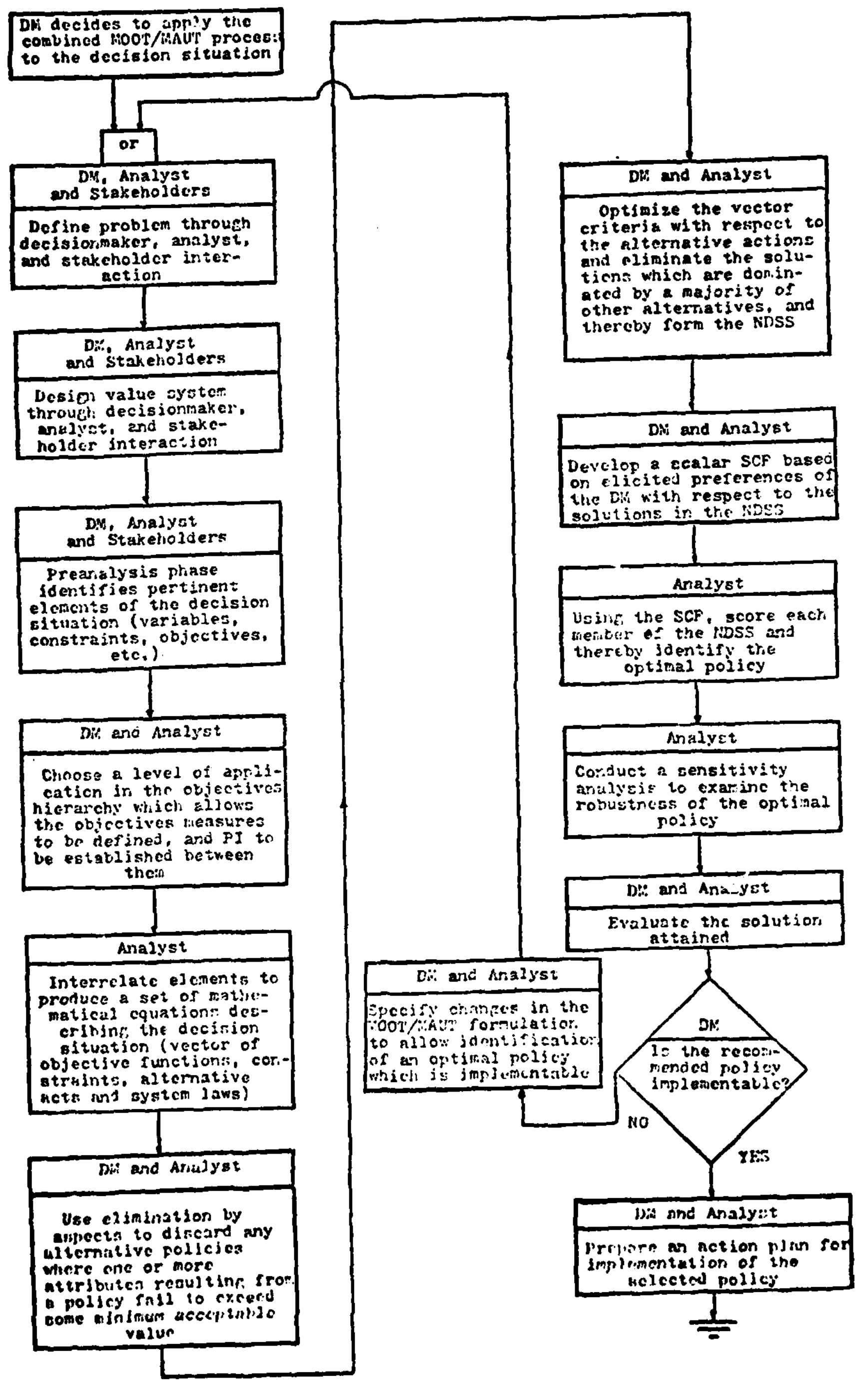

Figure 2. DELTA Chart For The Combined MOOT/MAUT Process (Deterministic/Static Case)

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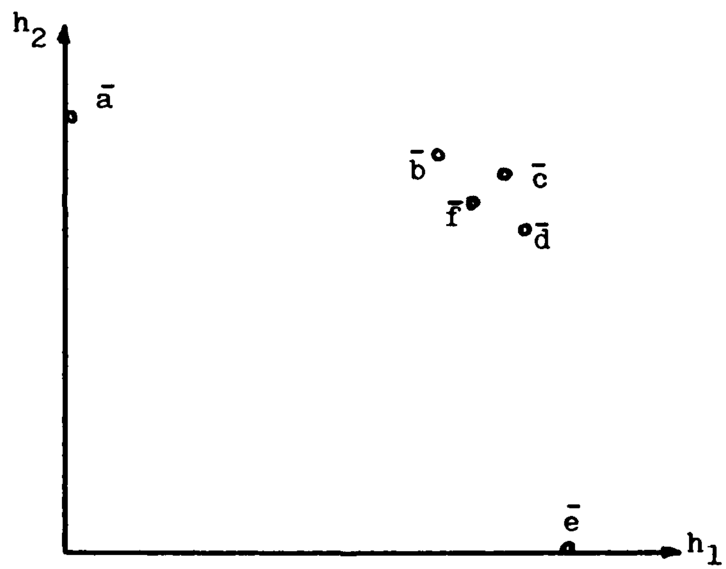
using some version of lexicographic ordering or elimination by aspects. The elimination by aspects procedure (Tversky, 1971) compares the alternatives by looking at specific attributes, and eliminates alternative policies which do not meet minimum requirements for one or more aspects of decision situation resolution. The analyst continues this refinement of alternatives with the DM until an appropriate set of alternative policies is produced. Elimination by aspects usually requires that the attainment levels of the attributes are continually made more restrictive until only one alternative remains. In our approach, a set of minimum attainment levels is obtained and if more than one alternative remains, other means as described below are used to select the optimum alternative. This generally allows a successful tradeoff between flexibility, time, and tractability.

d. Optimization of the multiple objective functions is then accomplished. Optimization of the multiple objective functions is required because the scalar SCF has not yet been formed (which would require optimization/ranking of alternatives only with respect to a scalar). In the optimization process, the analyst uses one of the mathematical optimization techniques to refine or tune the decision variables in each alternative policy to produce the best candidate forms of each alternative policy. Using these robust forms of mathematical optimization allows the model to deal explicitly with the notion of time and to show precisely the impacts from the policies. The implementation of this optimization process is discussed in Section 2.1 of Chapter 2.

e. The elimination of dominated alternatives is accomplished to reduce the decision space to only those alternative policies which have attribute levels which are non-dominated, bearing in mind the caveat discussed previously concerning elimination of an alternative which is not dominated by a majority of other alternatives. The efficient set of the best candidate forms of the alternative policies then forms the NDSS. In some cases, it may be reasonable to form a feasible essentially NDSS. For instance, Figure 3 shows a set of solutions for the two attribute case. The strict NDSS is composed of solutions $\bar{a}$, $\bar{b}$, $\bar{c}$,

Feasible Solution Set $(\bar{a},\bar{b},\bar{c},\bar{d},\bar{e},\bar{f})$
Dominated Solution Set $(\bar{f})$
Non-Dominated Solution Set $(\bar{a},\bar{b},\bar{c},\bar{d},\bar{e})$
Feasible Essentially NDSS $(\bar{b},\bar{c},\bar{d},\bar{f})$



Attribute Vector $h^T=(h_1,h_2)$

Figure 3. Feasible Essentially NDSS

$\bar{d}$, $\bar{e}$. But for a DM who values both attributes, solutions $\bar{a}$ and $\bar{e}$ are not very good (although they are non-dominated), and solution $\bar{f}$ is very good (although it is dominated). In this case, it would seem reasonable to form a feasible essentially NDSS composed of solutions $\bar{b}$, $\bar{c}$, $\bar{d}$, $\bar{f}$ which should be examined closer for selection of the optimum.

f. The policy selection phase is a MAUT exercise where the preferences of the DM are elicited to form a scalar SCF. This process is discussed in Chapter 3 as well as described by Keeney and Raiffa (1976), and Dyer and Sarin (1978). This SCF then becomes the scoring criterion by which the members of the NDSS are evaluated and ranked. The optimal policy is then identified and presented to the DM for consideration.

g. An analysis of results is conducted to evaluate the sensitivity of the solution to variation in parameters. Critical parameters are identified, and an improvement in the accuracy of these parameters is attempted. The final step in the algorithm is planning for implementation of the optimum policy.

This algorithm is iteratively applied until no significant change occurs in the results. Overall, this algorithm which allows each of MOOT and MAUT to be used in individually specialized but complementary steps is more efficient than resolving the decision situation with either approach alone. Modifications to this algorithm due to uncertainty of outcome and time varying relationships are discussed for the various parts of the algorithm in Chapters 2 and 3. Likewise, the general input requirements and output results for the MOOT/MAUT approach are summarized under the MOOT and MAUT descriptions in Chapters 2 and 3.

The benefits of the combined MOOT/MAUT approach compared to either process are as follows: there is more efficient resolution of decision situations because of the complementary nature of the combined processes (accomplish the required optimization with the technique designed for efficient optimization--MOOT, and accomplish the required ranking of alternatives and decision making with the technique designed for this purpose--MAUT); this combined approach can be used if there is limited access to the DM because the interview time can be partitioned into elicitation of alternate act scores or utilities, and criterion or

attribute weights elicitation, while allowing the analyst to continue in the modelling and optimization steps between these elicitations. The main cost of the combined MOOT/MAUT approach compared to either approach alone is an expected increase in computer resources utilized particularly as the number of attributes increases. Table 1 includes comments on the costs and benefits of the approaches.

4. An Example Of The MOOT/MAUT Process

The following is an example to illustrate the combined MOOT/MAUT process. This example is very similar in form to the MOOT process example (Chapter 2) because of our inclusion of a choice function based on utility theory in the decision making step of that example. This example will also be from the area of welfare economics.

Welfare economics is that branch of economics which deals with the distribution and consumption of resources for the public good as opposed to the individual good. The general objective of welfare economic analysis is the evaluation of economic alternatives and redistribution of economic resources for maximum societal benefits. An allocation of resources in Pareto optimal when no other reallocation of production and distribution will increase the economic satisfaction of any one individual without decreasing the satisfaction level of others in society.

Consider the following formulation of society's economic problem. For concreteness, we shall assume a simple closed economy with two consumers. We will assume that the economy is endowed with two factors, capital C and labor L. There are two outputs from production, a (clothing) and b (food). There are fixed endowments of labor and capital $\overline{C}$ and $\overline{L}$ described by

$$\overline{C} = C_a + C_b \quad (1)$$

$$\overline{L} = L_a + L_b \quad (2)$$

where $\overline{C}$ and $\overline{L}$ are the maximum levels of factor supplies, C_a and C_b are the amount of capital allocated for producing a and b respectively, and L_a and L_b indicates the amount of labor which is allocated for producing

Table 1

Comments on Multiple Criteria Approaches

Approach	Cost of Using	Benefit of Using
MOOT	DM may not feel involved in certain aspects of the process such as the optimization and formation of NDSS	Presents the candidates for optimal alternative in the form of a NDSS
MAUT	Requires detailed elicitation of the DM's preference structure	Ranks all alternatives in terms of scalar performance index
MOOT/MAUT	Generally requires more analysis time (and computer time) because of the comprehensive approach	A comprehensive approach that incorporates the DM into several phases which should increase the acceptance of this approach by DMs. May require less total DM involvement then either process used separately.

a and b respectively. The production functions which determine the amount of clothing and food produced are given by

$$a = f_i \; (C_a, L_a) \tag{3}$$

$$b = f_j \; (C_b, L_b) \tag{4}$$

Two alternative actions (A_1, A_2), which are each composed of different production functions, have been defined. Each alternative action is composed of the four policy variables C_a, C_b, L_a, L_b. In this example, not only will the optimum alternative which maximizes the utility of the two consumers be identified, but also the best levels of the policy variables for the optimum will be found.

We assume that all agricultural and clothing production will be distributed between the two consumers as needed to maximize their happiness. Thus we have

$$a = a_1 + a_2 \tag{5}$$

$$b = b_1 + b_2 \tag{6}$$

where a_1 and a_2 are the amount of clothes allocated to consumers 1 and 2 respectively, and b_1 and b_2 are the amount of food allocated to consumers 1 and 2 respectively. The objective of each consumer is to maximize his or her utility.

We assume that the utility of each consumer depends directly on the quantity of products consumed. The utility functions for the consumers are assumed to be the isotone functions

$$u_1 = g_1 \; (a_1, b_1) \tag{7}$$

$$u_2 = g_2 \; (a_2, b_2) \tag{8}$$

The vector of objectives functions is defined as

$$\text{Max } J = \text{Max } (u_1, u_2) \tag{9}$$

An elimination by aspects effort is conducted by setting minimum attainment levels of the attributes to determine if any alternatives or forms of alternatives should be eliminated from consideration. This process will usually reduce the decision space, and hence one must be careful to justify the minimum attainment levels so as to prevent

elimination of an alternative on a single unfounded aspect. The optimization step commences with the optimization of one objective function while holding the value of the other objective function constant.

Now we can pose the above problem as

$$\text{Max} \quad u_1 \tag{10}$$

$$\text{subject to } u_2 = \bar{u}_2 \tag{11}$$

$$\bar{C} = C_a + C_b \tag{12}$$

$$\bar{L} = L_a + L_b \tag{13}$$

$$a = f_i\ (C_a, L_a) = a_1 + a_2 \tag{14}$$

$$b = f_j\ (C_b, L_b) = b_1 + b_2 \tag{15}$$

where $\bar{u}_2$ is a specific level of utility for consumer 2. Now adjoining the constraints to u_1 with Lagrange multipliers and optimizing yields optimal values for the policy variables.

The optimization problem is optimized many times for various values of $\bar{u}_2$ (effectively this is allowing the Lagrange multiplier to change values). Iterating the optimization procedure with each alternative for various values of u_2 will produce solutions which are Pareto optimal for each alternative. After the set of feasible optimal solutions has been generated, a digital computer sub-routine can be used to find the non-dominated solution set (NDSS). We will call the NDSS the set Y. A MAUT technique is used in the policy selection step where the preferences of the DM with respect to the attributes are used to construct a scalar social choice function (SCF). This SCF ($\hat{Y} = f\ (u_1, u_2)$) is used to select the best solution from among the NDSS. Assuming that difference independence (in addition to preferential independence) among the attributes holds (Dyer and Sarin, 1979), the form of the SCF is

$$\hat{Y} = \alpha \cdot u_1 + (1 - \alpha) \cdot u_2 \tag{16}$$

After the scaling constant, α, has been assessed (Dyer and Sarin, 1979), the specific solutions in Y (NDSS) are ranked by $\hat{Y}$ and the best solution is identified. This algorithm will score each member of the NDSS and produce a set of $\hat{Y}$ for the NDSS.

When maximum $\hat{Y} = \hat{Y}^*$ is found, this solution can be traced back to a specific action A_1 or A_2 (depending on whether $\hat{Y}^*$ came from the A_1 or A_2 optimization process). This action (A_1 or A_2) can then be related to the optimum levels of the decision variables $\hat{C}_a$, $\hat{C}_b$, $\hat{L}_a$, and $\hat{L}_b$. The sought after-solution includes the identification of the optimum distribution of commodities $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_1$, and $\hat{b}_2$ which would result from the optimal policy. Iteration of the MOOT procedure along with a comprehensive sensitivity analysis gives the DM an idea of the robustness of the optimal policy. An action plan for implementation of the optimal policy would normally follow.

A numerical realization of the previous example will be presented before extending the example to the case of multiple DMs. Assume that a pre-analysis phase has established that the form of the objective functions for the two consumers is

$$u_1 = a_1 + b_1 \tag{17}$$

$$u_2 = (a_2\ b_2)^{.5} \tag{18}$$

and two alternative production systems are defined as follows:

Action A_1 (use production system 1)

$$a = f_1\ (C_a,\ L_a) = (C_a\ L_a)^{.5} \tag{19}$$

$$b = f_2\ (C_b,\ L_b) = (C_b\ L_b)^{.5} \tag{20}$$

Action A_2 (use production system 2)

$$a = f_3\ (C_a,\ L_a) = C_a - 1. \tag{21}$$

$$b = f_4\ (C_b,\ L_b) = (L_b)^{.5} - 2. \tag{22}$$

In this simplified example, we assume there is no risk or uncertainty. An elimination by aspects exercise establishes that no policy forms with $u_1 < 2.$ and $u_2 < 1.5$ will be allowed. In sequence, the equations from each alternative production system along with the objective functions are substituted into equations 10, 11, 14 and 15 and the optimization process is carried out. Now each value function is optimized for both alternative systems. The solutions for the proposed alternative systems 1 and 2 and the resulting NDSS are shown on Figure 4. Assume

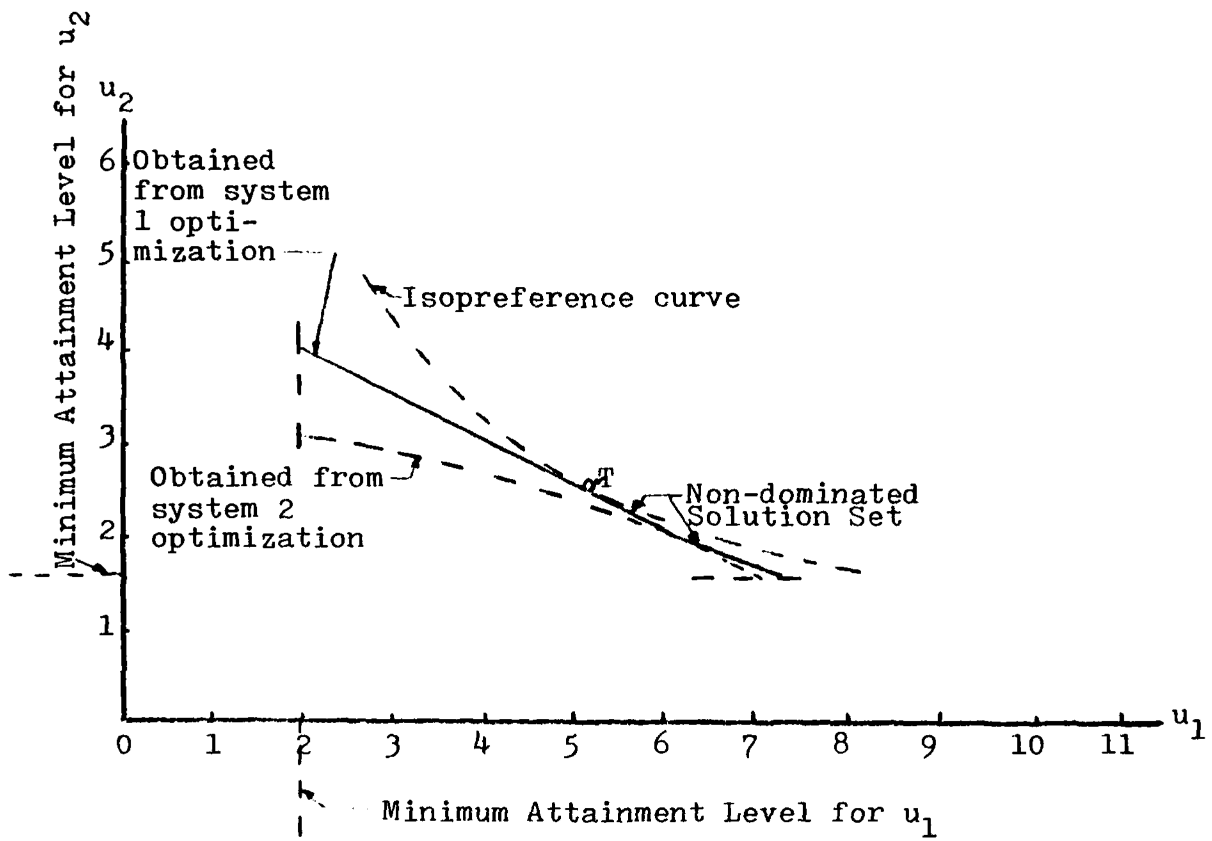


Figure 4. Optimal Policy Selection

that the attributes are preference independent, and that the SCF has been determined to be

$$\hat{Y} = v\, u_1\, u_2 \tag{23}$$

where v is a constant determining a family of isopreference curves. This SCF is tangent to the NDSS at $v = 0.08$ at point T in Figure 4. Point T is then the optimum policy of implementing production system $1(A_1)$ with 5.0 units of capital ($\hat{C}_a$) and 5.0 units of labor ($\hat{L}_a$) going into production of clothing and 5.0 units of capital ($\hat{C}_b$) and 5.0 units of labor ($\hat{L}_b$) going into production of food. This policy will produce the allocation of 2.5 units of clothes ($\hat{a}_1$) and 2.5 units of food ($\hat{b}_1$) to consumer 1 ($\hat{u}_1 = 5$), and 2.5 units of clothes ($\hat{a}_2$) and 2.5 units of ($\hat{b}_2$) food to consumer 2 ($\hat{u}_2 = 2.5$).

A sensitivity analysis would give feedback as to the validity of this consensus opinion before implementation is actually planned. In this sensitivity analysis we would include an investigation of the impact on optimal policy identification that the variation in scaling parameter values cause.

The deviation required in the MOOT/MAUT process for multiple DMs is similar to those for either the MOOT or MAUT process. These deviations from the processes for a single DM are described in Section 4 of Chapters 2 and 3.

5. Summary

In this chapter, both MOOT and MAUT processes were compared and contrasted with respect to structure, function, purpose and resulting organizational implications. While there are operational and philosophical differences between MOOT and MAUT, both approaches are mental constructs to approaching decision situations and when compared at the same level, there are for all intents and purposes, no differences in the structure between them. As demonstrated in examples and confirmed in Appendix B, both approaches are capable of producing either identical or strategically equivalent policies when applied at the same level to a common decision situation. The comparison shows that MOOT is adept at solving hardware oriented (elements are quantifiable) parameter design

problems which may be time dependent, while MAUT is well suited for a wide range of application including those with behavioral aspects (including qualitative elements). Next, an investigation of the organizational implications of use of both approaches produced the observation that the only generally significant variant with respect to information, cost, and time requirements of either process is the flexibility in the MOOT process which allows the contact time to be divided between analyst and DM into several elicitation sessions without unnecessarily delaying the process.

As one examines MOOT and MAUT at the practice level, a certain amount of mixing of both approaches becomes apparent. Therefore, it is logical to combine both approaches into a common multiple criteria synergistic approach which takes advantage of their complementary nature for better resolution of decision situations. This combined approach allows portions of each of MOOT and MAUT to be used efficiently in an appropriate step of the systems engineering methodology. A DELTA chart (Figure 1) is presented to aid decision makers and analysts in choosing the correct approach to use in modelling and resolving a large-scale decision situation.

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CHAPTER 5

AN APPLICATION OF MULTIPLE CRITERIA DECISION THEORY TO ELECTRONIC WARFARE AIRCRAFT RETROFIT DESIGN

1. Introduction

The retrofit of a particular aircraft, with equipment designed for a mission the aircraft was not originally intended to fly, is a complex and time consuming process. When U.S. Air Force requirements for a special purpose aircraft (electronic warfare, reconnaissance, etc.) are developed, there is generally a concerted effort, for economic reasons and reasons of time to completion of effort, to modify an existing airframe as opposed to designing an entirely new one. Difficulty arises concerning how to fit a wide variety of equipment into an airframe and also satisfy all concerned parties with respect to cost, performance, and schedule (OMB 1976; USAF, 1976b). Previous efforts at Electronic Warfare Aircraft Retrofit Design (EWARD) have met with limited success (Peterson, et al., 1975; Cook, 1977). The basic problem is that the retrofit aircraft is often not what the users originally asked for or need. Instead, the retrofit generally results in an aircraft which often does not sufficiently ameliorate deficiencies which led to the design requirement. A combination of budgetary, political and technical factors often leads to system development delays. This often results in a system being developed in a later time frame than the one which it was needed and the one in which design requirements were specified.

Current directives (OMB, 1976; DOD, 1977a; USAF, 1977a; USAF, 1976a) stress the incorporation of the systems oriented approach and evaluation criteria concerning performance, cost and schedule. As stated in DODD 5000.1: "System development shall be continuously evaluated against these requirements (programs and equipment which exhibit

timely development and high performance at a minimum cost) with the same rigor as that applied to technical requirements. Practical tradeoffs shall be made between system capability, cost and schedule." There are many obstacles to implementing the spirit of these current directives as will be described in Section 2.

The purpose of this effort is to illustrate how multiple criterion decision theory (MCDT) can be applied to the specific application area of an initial phase of the DOD Equipment Acquisition Cycle for an electronic warfare aircraft retrofit design (EWARD). A combined multiple objective optimization theory/multiple attribute utility theory (MOOT/MAUT) approach was applied to EWARD to investigate:

1. If a multiple criteria decision theory (MCDT) approach can improve the EWARD process.

2. If application of a combined approach using both MOOT and MAUT has merit.

3. If an adequate set of criteria can be generated to judge the goodness of alternative designs early in the EWARD process (Table 1)

The retrofitting of a special purpose EW aircraft is a large-scale system problem because of the political, military, economic, and technical overtones. EWARD requirements are therefore appropriated candidates for an approach using systems engineering methodology and MCDT within this methodology. The first three steps of problem definition, value system design, and system synthesis were accomplished (Section 3), and then a combined MOOT/MAUT approach was used to perform the next steps (modelling and systems analysis, ranking alternatives/decision making) as described in Section 4. The solution obtained from this approach to EWARD was validated on an EW aircraft now in operation using appropriate data and Government advisors and decision makers (DMs).

Table 1

Criteria for an EW Retrofit System

1. Technical: EW Aircraft Aerodynamic Performance
    a. EW System Weight
    b. EW System Volume Required
    c. EW System Power Required

2. Economic: EW Retrofit System Life Cycle Cost

3. Military: Retrofit System Electronic Warfare Performance
    a. Aircrew Performance
    b. Number of Threats Degraded
    c. Number of Threats Defeated

4. Political: National Policy Satisfaction

2. The Electronic Warfare Aircraft Retrofit Decision Situation

The retrofitting of equipment to satisfy a particular need is not a new concept in military or civilian history. The size and complexity of current retrofit efforts in the EWARD situation in the U.S. and the associated political, economic, military and technical impacts make this a large-scale system problem. The economic concerns are felt from the emphasis of the U.S. Congress and upper echelon military policy makers who must consider budgetary constraints and alternate program trade-offs. The size of the problem is significant when one considers that a fleet of fully equipped EW aircraft can cost ten times the price of the original aircraft. The political factors are also considerable when the governmental policy makers consider the ramifications of putting certain equipment on the aircraft which impact U.S. security, NATO agreements, SALT talks, foreign weapons sales, etc. The technological concerns, while easier to quantify, are nevertheless substantial when one considers the problem of fitting sophisticated electronic equipment into an airframe designed primarily to carry ordinance. The problem of size, weight, volume, antennas, power type, air crew requirements, etc. must be considered (Cook, 1977 ; Peterson, et al., 1975; USAF, 1977a). Optimization techniques which consider only the technical aspects of the retrofit design problem have met with only limited success (Peterson, et al., 1975).

To give an appropriate perspective, from which the difficulty in the retrofit design of an EW aircraft is apparent, the prescribed retrofit procedure will be presented along with a set of impediments to enactment of this process. Figure 1 shows the phases of an EW retrofit system program as viewed by government program managers. The five phases of Conceptual, Validation, Development, Production, and Deployment contain various funding decision points. The stakeholders are amalgamated into three groups. Group G-1 (Operations and Intelligence) is composed of the system users (using commands, Strategic Air Command - SAC, Tactical Air Command - TAC, etc.) and the intelligence community. Group G-2 (Government policy) is made up of Headquarters U.S. Air Force (Hq. USAF), Congress, and the executive branch. Group G-3 (Technical

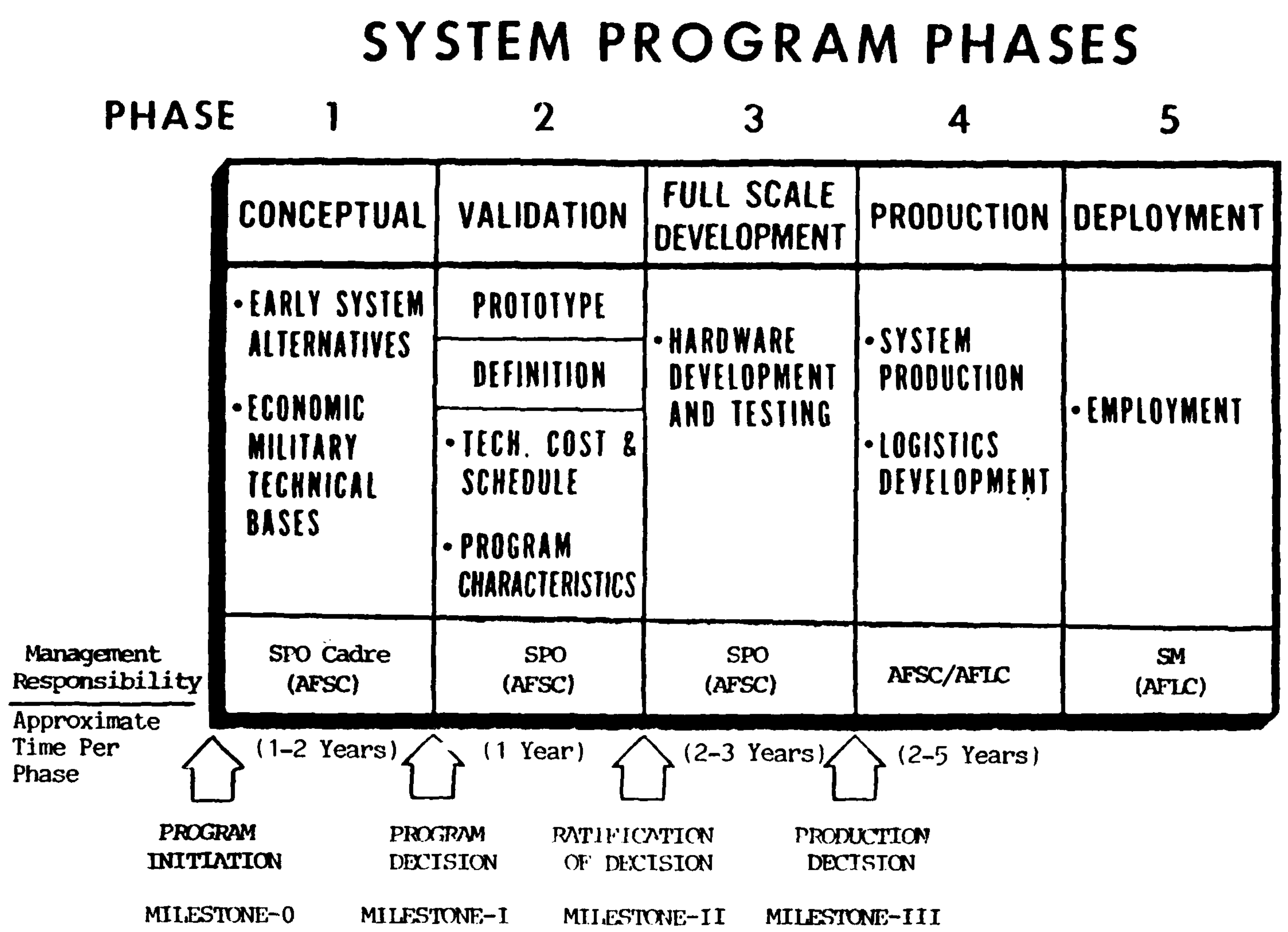


Figure 1. Defense Systems Equipment Program Phases

development and assessment) is made up of the industrial contractors, and in-house government development, contract monitoring and user interfacing subgroups (Air Force Systems Command - AFSC, and Air Force Logistics Command - AFLC). Decision makers and advisors from these three groups, who are involved in EW equipment acquisition, took part in this effort.

2.1 Specified EW Retrofit Procedure

The USAF is guided in the procurement of military equipment by various regulations and directives (eg., DOD, 1977a; USAF, 1966b). The following discussion describes the prescribed way that an EW retrofit is accomplished in the U.S. Air Force. The process starts with the identification by the using commands in group G-1, or the intelligence community, of a deficiency or need. Figure 2 illustrates the process. This deficiency can be a previously recognized weakness which now can be corrected through successful efforts of government laboratories or industrial contractors through the acquisition of a new system. This deficiency or need is presented by the using command to Headquarters Air Force (Hq. USAF) in the form of a statement of need (SON), and this is where group G-2 becomes involved (USAF, 1978). If the initial estimate of system research, development, test, and evaluation (RDT & E) exceeds $75 million or $300 million in production, the program is designated a major systems acquisition. As a major systems acquisition, the Defense Systems Acquisition Review Council (DSARC) review program is required. The Mission Element Need Statement (MENS) is next generated which must identify the mission need in terms of the task to be performed, assessment of projected enemy threat, and existing DOD capability (DOD, 1977b). Hq. USAF reviews the MENS and forwards it to the Secretary of the Air Force (SAF) who approves it and sends it to the Secretary of Defense (SECDEF) for final approval, or redirects it for appropriate modification or termination. If the need is judged as legitimate and current by the SECDEF, the program is initiated (milestone 0) by authorization of funds for the Conceptual Phase.

A DELTA chart documenting the Conceptual Phase is illustrated in

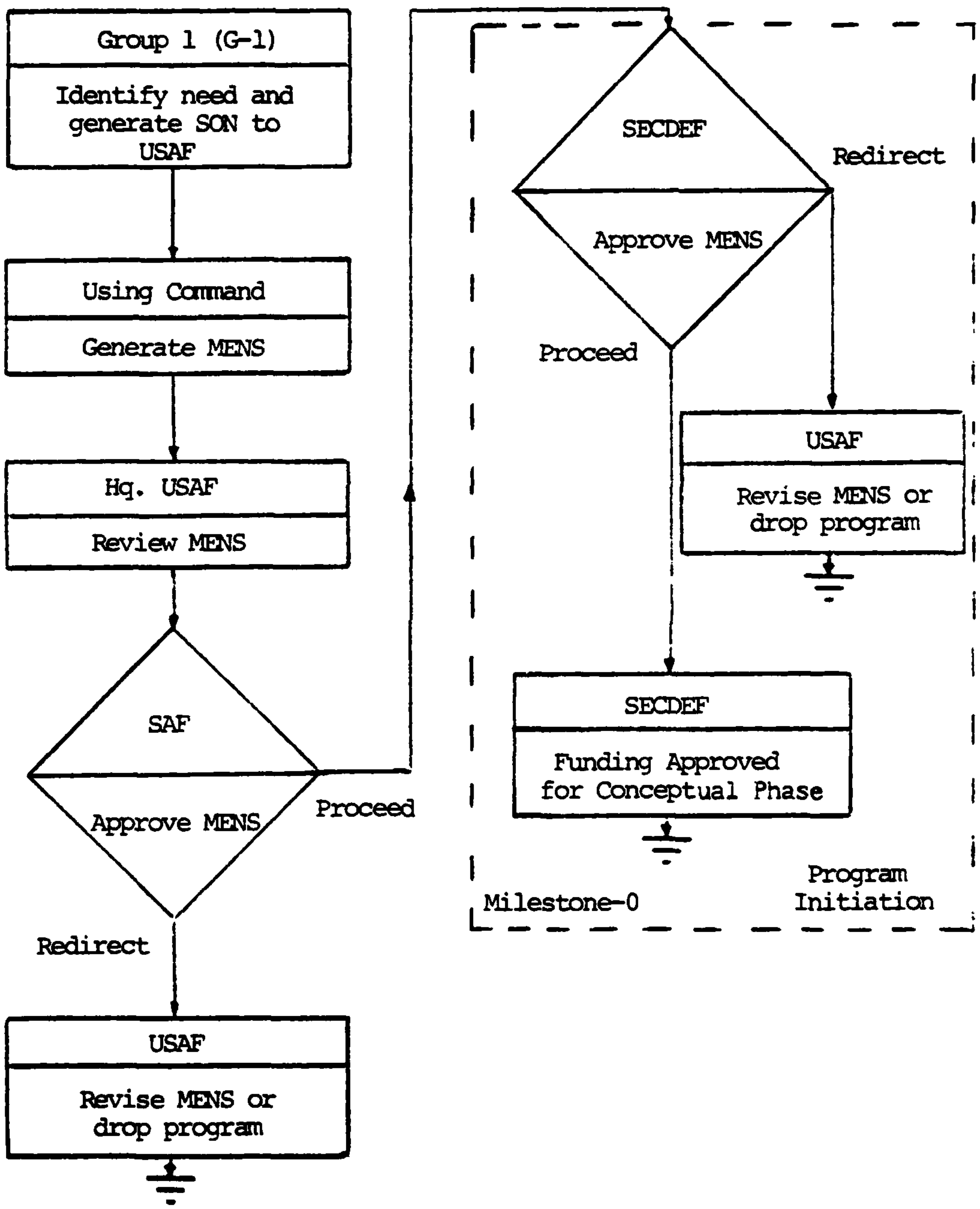


Figure 2. Defense Systems Equipment Acquisition Program Initiation

Figure 3. Funding is made available to a System Program Office (SPO) cadre in group G-3 to define the acquisition problem, identify program objectives and goals, and alternative candidate systems. The SPO also develops models to evaluate operational considerations, acquisition approaches and associated risk factors. Using cost and performance trade-offs, candidate systems are evaluated to identify one or more alternatives for entry into the Validation Phase. Next, development of a Program Management Plan (PMP) is undertaken as the summary of the previous efforts. The PMP is used as the basic document defining pertinent aspects of the retrofit system. The PMP is used to prepare the Program Management Directive (PMD) which summarizes the previous efforts in the Conceptual Phase, and presents a plan for proceeding into the Validation Phase. Hq. USAF uses the PMD to generate the Decision Coordinating Paper (DCP) as input to the Air Force Systems Acquisition Review Council (AFSARC). AFSARC makes recommendations on the program and forwards these to the SAF. If the DCP is approved, it is passed to the DSARC (milestone I) for action. Following recommendations by DSARC, the SECDEF is tasked with final decision on the program. If approval is granted, funding authorizes proceeding into the Validation Phase. The Conceptual Phase is purely a "paper" effort with no funding authorized for hardware.

The Validation Phase is illustrated in Figure 4. When the Validation Phase is authorized by the SECDEF, a SPO (G-3) is tasked with generating the basis from which one or more contractors are selected to go into the Development Phase. Validation is achieved through either a contract definition (paper design) or a prototype (hardware demonstration) approach. In the "contract definition" approach, usually two (or more) contractors are allowed to compete with each other in an attempt to further define and refine the system. A Request for Proposal (RFP) is issued which initiates the paper study. The results of this phase are system specifications and a statement of work. A source selection team, including representatives from G-1, G-2 and G-3, selects the most attractive contractors from the competing group. A RFP is issued and funding negotiations for the Development Phase are completed with the selected

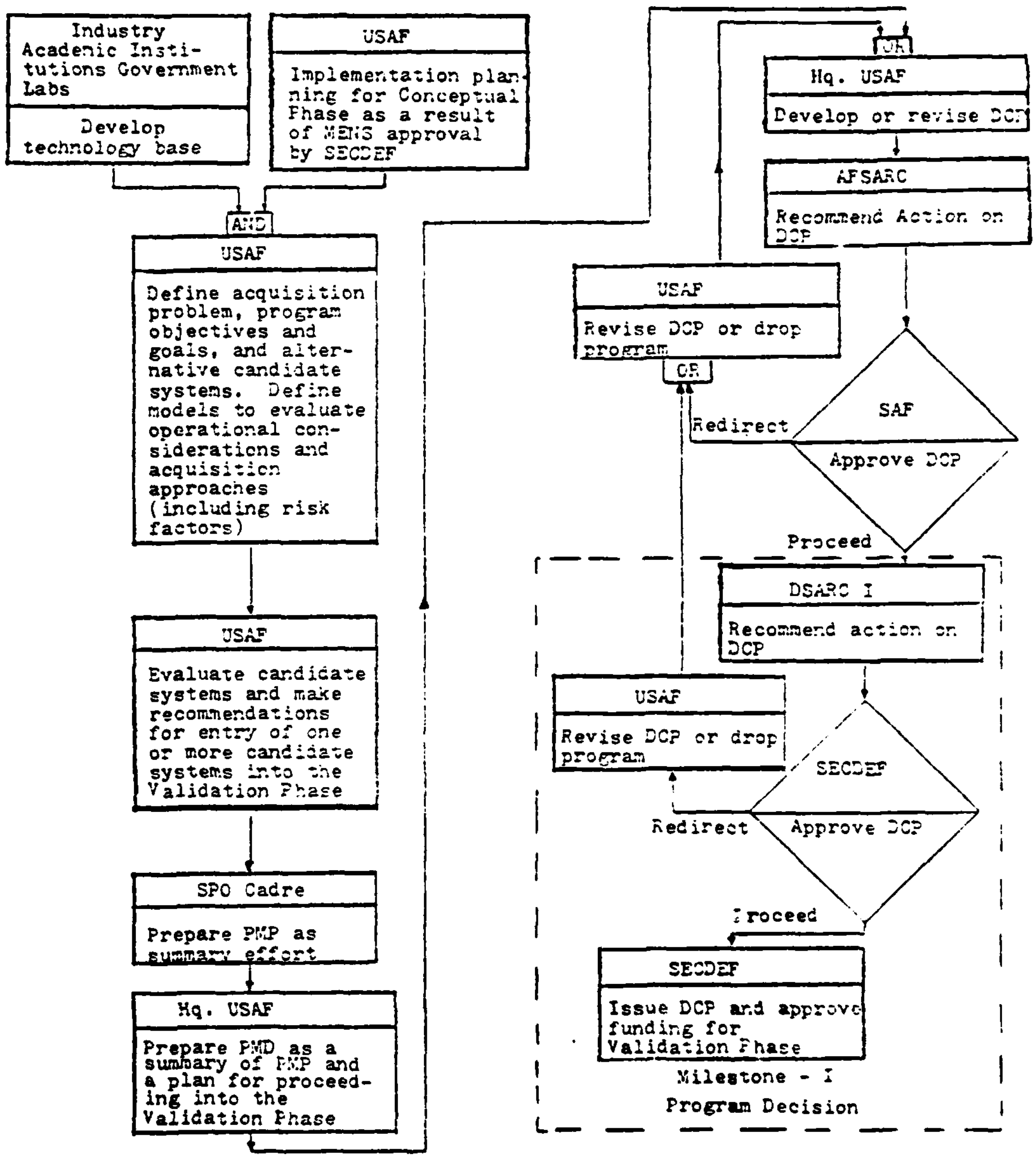


Figure 3 Defense Systems Equipment Acquisition Conceptual Phase

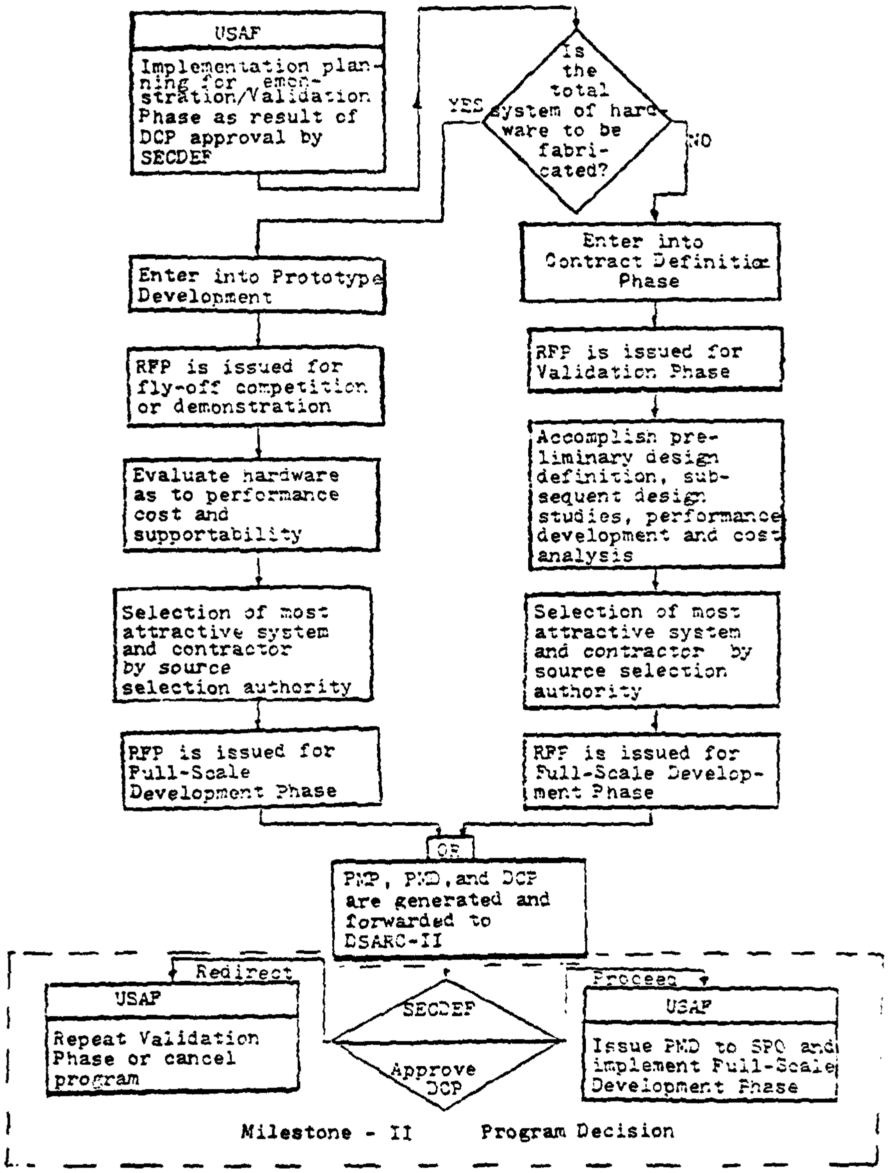


Figure 4. Defense System Equipment Acquisition Validation Phase

contractors. In the "prototype" approach, a Development Concept Paper (DCP) from the Service Secretary (G-2) initiates the process. A formal RFP is distributed to industry, and a Source Selection Team usually chooses one or more contractors to continue as a result of the submitted proposals. The selected contractors fabricate a hardware version of the system under development. This hardware system is evaluated analytically in a demonstration or "fly-off" exercise. During this evaluation, a RFP is prepared for Full-Scale Development, and the most satisfactory competitor is selected for further development. In either contract definition or prototype approaches, a PMP is prepared next, followed by the DCP and PMD, and a DSARC board meets for milestone II (G-1, G-2, G-3) to judge the worthiness of the program to proceed. If the program is judged essential and proceeding satisfactorily, the SECDEF acts on the program. If the program is approved, a PMD (G-2) is sent to the SPO and funding is approved as authorization to proceed to the Full Scale Development Phase. Other alternative actions to proceeding into Development are to return to more validation, or cancellation.

The Full-Scale Development Phase provides the expanded engineering design, fabrication, testing, evaluation, and support planning for the selected system. The "user" and "supporting commands" participates in the Development Test and Evaluation (DT & E) and Initial Operational Test and Evaluation (IOT & E). The contractor negotiates for production during the testing process, and configuration audits (FCA and PCA) are accomplished subsequent to finalizing the system configuration. After this, any change in the system is rigidly controlled and must follow the formal Engineering Change Proposal (ECP) route. The results of the Development Phase are presented to DSARC at milestone III for review. If approval is granted and Office of the Secretary of Defense (OSD) funding procured, the program enters production.

In the Production Phase, the system is produced by the contractors and logistic support is procured. This by far is the most costly and time consuming phase up to this point.

The completed system is turned over to the user in the Deployment Phase by the Systems Manager (SM) in Logistics Command (AFLC). There

the system is utilized and maintained until its retirement. Figure 1 shows the major phases in the idealized system life cycle and the approximate time for completion of each phase.

2.2 EW Program Complications

The process just described for EW retrofit of an aircraft is seldom followed exactly because of a number of complex factors pertinent to electronic warfare. There are seven basic reasons for EW retrofit difficulties.

1. Electronic Warfare is a highly technological, expensive, and specialized business. EW equipment requires extensive dedicated research and development capabilities that only a limited number of industrial contractors have established. The risks in developing and retrofitting these sophisticated and specialized systems are high, and the spin-offs to commercial application are severely limited.

2. There is insufficient communication between all stakeholders at all phases of the system cycle. There is a lack of effective interchange of information between groups G-1, G-2, and G-3 in the Conceptual Phase of system development. An exception to this lack of communication occasionally exists between the upper level DMs in groups G-1 and G-2 when politically sensitive equipment is involved. The general lack of communication makes any kind of long range planning for the system retrofit very difficult. This lack of communication prevents the cost and performance people from coming to early agreement which generally means time delays at later phases in the system cycle.

3. The decision making structure is multilevel and semidefined. The decision makers (DMs) and their advisors in G-1 and G-3 groups are defined but arranged in multilevels which makes it difficult for amalgamation of objectives at these various levels. The DMs and their advisors in G-2 group are also arranged in multilevels but are not clearly defined. This means that certain gerents can participate in varying degrees in their decision making role depending on factors such as the political atmosphere. This makes it particularly difficult to account for some DM's interactions.

4. Government policy makers do not operate in sufficient isolation from private industry. The government policy advisors in G-1 and G-2 groups perpetuate a long standing amenitable relationship between themselves and EW industries. While this relationship can be beneficial to the government in certain aspects of contract negotiations, it can cause difficulties such as the fact that system deficiencies (and their amelioration) are often pointed out by the system builder or contractor instead of the intelligence community.

5. Long range government policy is difficult to forecast. The complex issues that affect foreign policy coupled with a bureaucracy that administers it, makes it particularly difficult to estimate accurately what the U.S. foreign policy will be for other than very short planning horizons.

6. The current funding directives (OMBC-109) encourage (and occasionally specify) dual-contractor development procedure for newly designed equipment. This is done as a way of insuring commonality in technology, and preventing a sole source supplier of replacement parts. Unfortunately this practice of carrying two contractors throughout the program also tends to cause funding and scheduling problems.

7. The contractor and retrofit program are often given flexibility with respect to cost and schedule commitments. The primary reason a program gets limited in scope, indefinitely delayed, or cancelled is that it has been surpassed on the priority list (and another program took its funds). The logical reasons for the above actions (lack of performance in the system, cost and schedule overruns) are not considered as prominently.

These factors presented above make the normal EW retrofit procedure (Section 2.1) difficult to implement. These items point out the need for a comprehensive approach to the EWARD such as supplied by MOOT or MAUT through the systems engineering methodology. These multiple criteria approaches allow the incorporation of a set of salient attributes in a way that allows one to address the requirements by individually considering factors which are affected by the impediments discussed previously. This flexibility is of significant value in a large-scale

effort like EWARD. In EWARD, the need exists for an adequate set of criteria which can be utilized in the evaluation of alternatives. The development and subsequent incorporation of these criteria into the difficulties cited produce a cost effective product that will meet the needs of the users.

3. Structure of the EWARD Decision Situation

In order to set the stage for the application of techniques from both MOOT and MAUT, a common pre-analysis effort covering the problem definition, value system design, and system synthesis phases was performed in order to identify and relate the factors of the U.S. Air Force EWARD structure. Sections 1 and 2 basically describe the pertinent stakeholders. The G-1 group (Operations and Intelligence) is responsible to point out deficiencies and coordinate requirements so the retrofitted system is operationally satisfactory. This group includes the eventual users of a retrofitted system. The G-2 group (Government policy) is the group that coordinates the systems impact on defense capability and foreign policy. This group also constrains the program with respect to budgetary considerations. The G-3 group (Technical development and assessment) defines the system configuration, and carries out and manages the research, development and production of the retrofit system. While other stakeholders are involved in EWARD, their interactions are extremely difficult if not impossible to define, assess or forecast.

The general needs of an EWARD are defined in the SON and MENS documents which delineate currently existing deficiencies. Historically, specific needs of the new system are described as threats to be countered in terms of requirements and EW techniques. The aircraft which will be retrofitted is often an obvious choice because of the performance requirements, operational situation, and current aircraft inventory supply. The number of these special purpose aircraft required for the specific mission is often specified and considered fixed in the early phases of the DOD equipment acquisition cycle. Therefore, the aircraft upon which the retrofit will be applied as well as the fleet size is hypothesized as determined,and its selection is not to be discussed in this

pre-analysis phase.

The possible options to ameliorate current deficiencies are listed in the Conceptual and Validation Phases in terms of passive procedures, transmitting and receiving system techniques, and active and passive expendables disbursement. The major constraints for the retrofit system are size, weight, power, and cooling restrictions (determined by the aircraft itself), funds available (a function of the deficiency priority and other factors), and time until deployment (a function of bureaucratic scheduling, and research and development - R and D status).

A subset of the objectives of the USAF is listed in Table 2. These upper level goals are arranged in the hierarchial intent structure of Figure 5. The main objectives of the EW task which were constructed as a result of interaction with the stakeholders are listed in Table 3. The lower level objectives of the EW task in their hierarchial order are shown in Figure 6. It is this latter set of objectives that were exploited in the solution of the EWARD. The measures by which attainment of the objectives is discerned (attributes) are shown in Figure 7 and listed in Table 4 for the lower level objectives. This set of attributes was used in the evaluation process for the alternatives in this EWARD effort.

While the objectives and their measures of attainment at the lower levels may seem self-explanatory, a brief description is provided to cover some of the aspects of the EWARD.

a. The aircraft weight attribute is a measure of the added weight (Kg) due to the total EW system plus a penalty figure (also in Kg) which accounts for any modifications to the fuselage caused by the EW system which would increase drag such as external antennas, external components, external wing pods, air induction cowlings for cooling, etc. The drag caused to the aircraft fuselage by a system is converted to equivalent penalty weight as a way of accurately quantifying the impact of a specific system on the aircraft performance without double counting with respect to another attribute.

b. The volume attribute (meters^3) is a measure of internal volume occupied by the EW system. The available volume for EW equipment is

Table 2.

Subset of Objectives of the USAF

* To Support the National Objectives and Policies of the United States

* To Maintain a Strong Air Force Capable of Multiple-levels of Response

* To Provide Forces Capable of a Strategic Response

* To Provide Forces Capable of a Tactical Response

* To Train and Maintain Forces to Support USAF Objectives

* To Perform the Reconnaissance Tasks Assigned

* To Perform the Electronic Warfare Task Assigned

* To Provide a Military Airlift Capability

* To Provide Support for Ground and Sea Forces

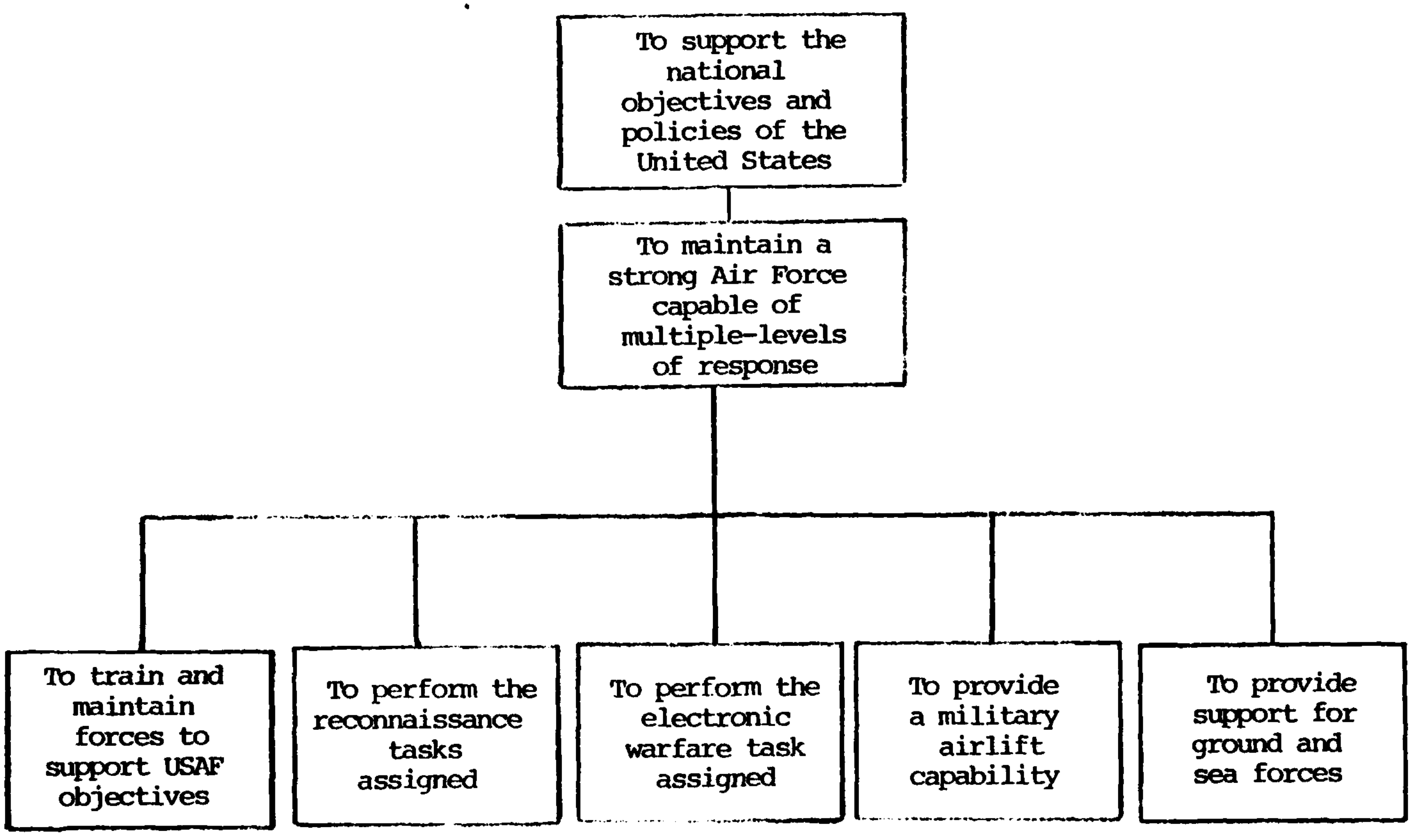


Figure 5. Hierarchy Subset of Intent Structure of USAF Objectives

Table 3

Electronic Warfare Objectives

* To Perform the Electronic Warfare Task Assigned
* To Provide for Electronic Support Measures (ESM)
* To Provide for Electronic Counter Measures (ECM)
* To Provide for Counter-Counter Measures (ECCM)
* To Maximize Aerodynamic Performance of EW Aircraft
* To Minimize Life Cycle Cost of the EW System
* To Maximize Electronic Warfare Performance of the Retrofit System
* To Maximize National Policy Satisfaction Through the EW System

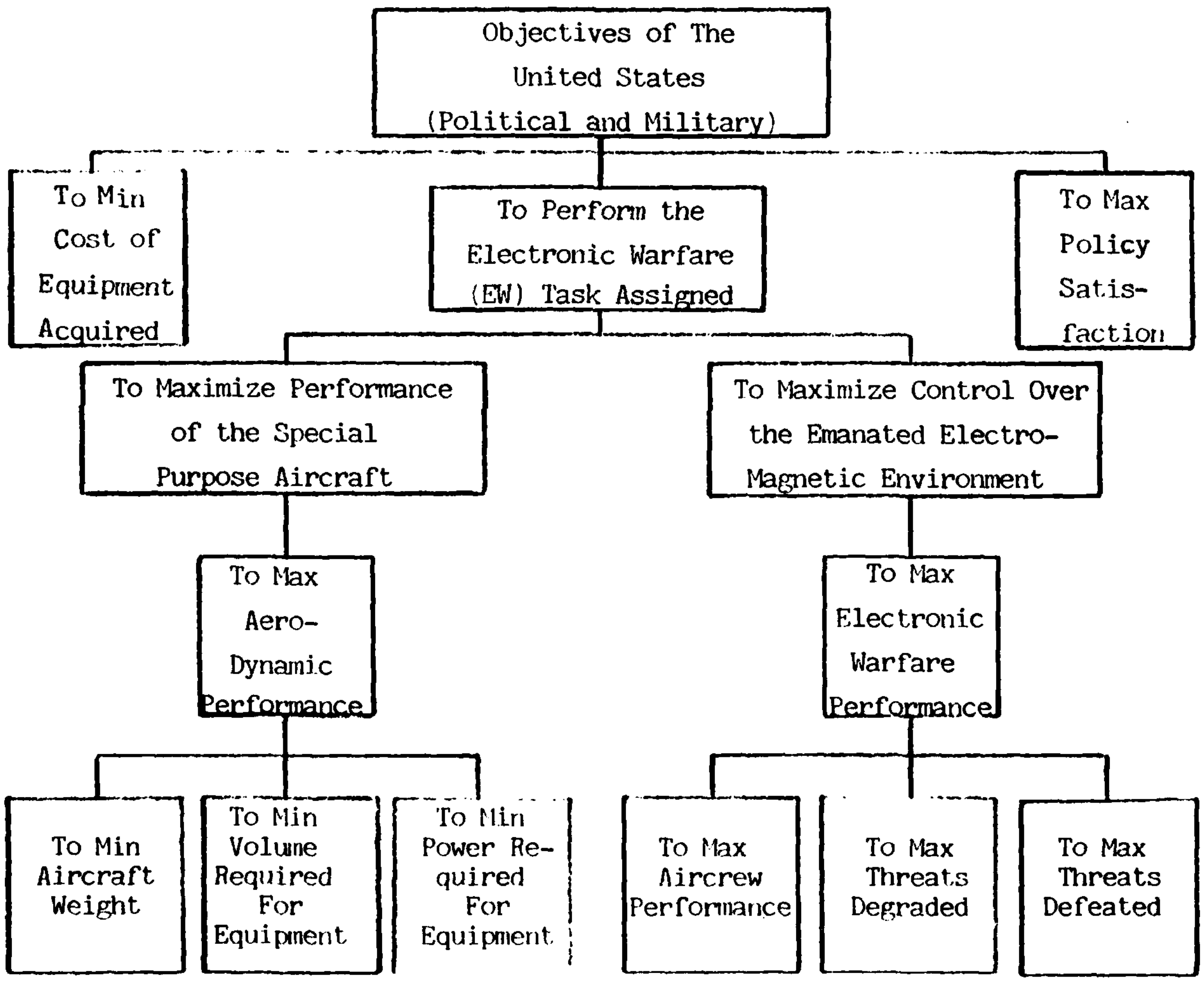
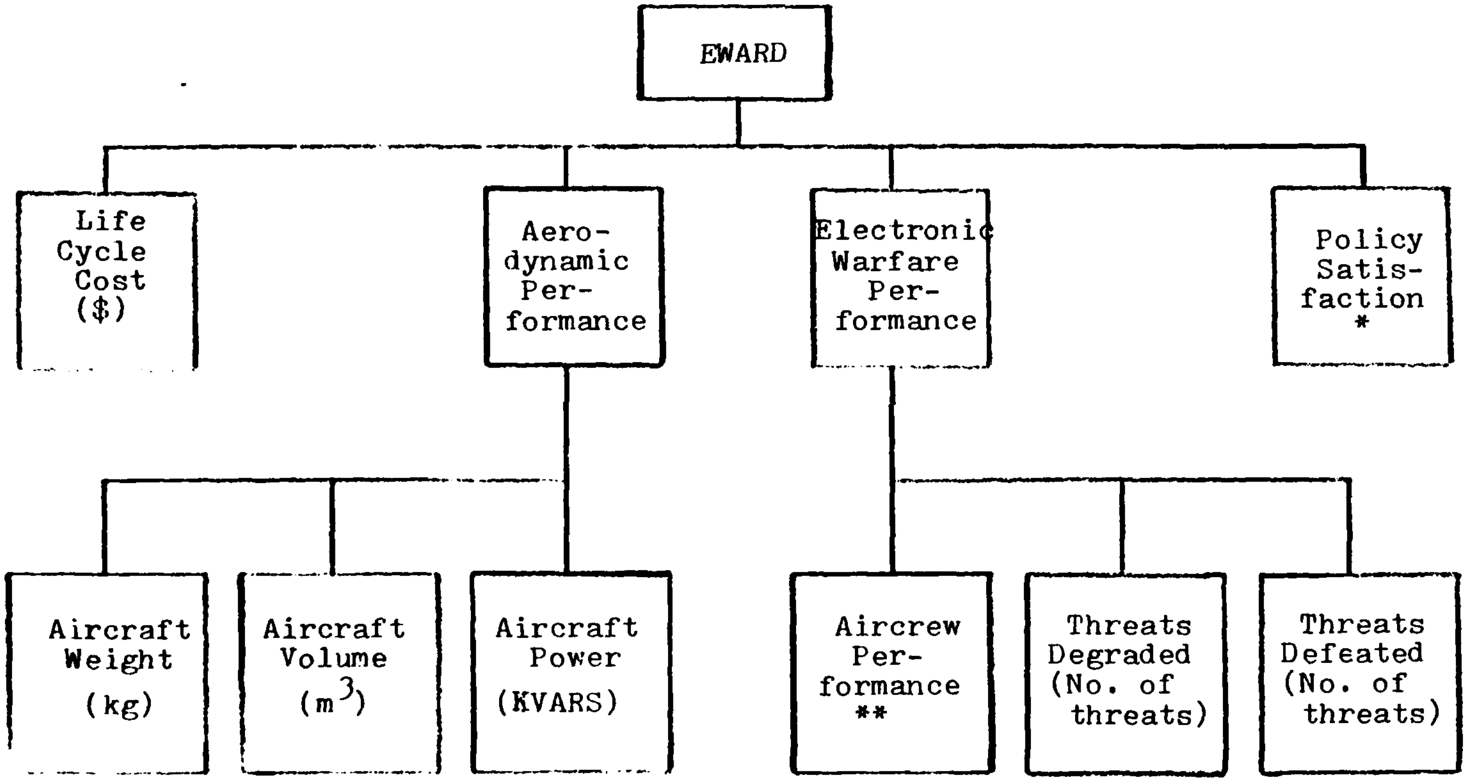


Figure 6. Hierarchy of Objectives of the EW Selection Process



** Direct Assessment
* Direct Performance Measure

Figure 7. Attribute Template For EWARD

Table 4

Lower Level Attribute Limits

Attribute	Greatest Level	Lowest Level
X_1 Aerodynamic performance		
X_{1a} EW system weight	4.5×10^3 kg	5.0×10^2 kg
X_{1b} EW system volume	4.10 m^3	1.5 m^3
X_{1c} EW system power	105.0 KVA	45.0 KVA
X_2 EW system life cycle cost	2.0×10^9 ($)	1.0×10^8 ($)
X_3 Electronic warfare performance		
X_{3a} Aircrew effectiveness	1.0 (normalized scale)	0.0
X_{3b} Number of threat types degraded	30.0	5.0
X_{3c} Number of threat types defeated	30.0	0.0
X_4 Degree of policy satisfaction	1.0 (normalized scale)	0.0

usually limited because of competition from other needed avionics equipment.

c. The power attribute (KVARS) is a measure of the peak electrical power required to operate the EW equipment (including auxiliary cooling).

d. The air crew performance attribute (direct performance measure) measures the degree of dedicated service that the EW system demands out of the crew for successful operation (some manual/automatic systems may require more crew operations than is physically possible). Values of this attribute can be obtained by simulation of the cybernetic candidate system. A normalized scale (limits of 0.0 to 1.0) was used to score the various alternative systems. The minimum score of 0.0 represents the case where the air crew was unable to perform the tasks required by an alternative system in a simulated combat environment. The maximum score of 1.0 represents the case where the air crew was able to perform all tasks required for completely successful operation of the alternative system in a simulated combat environment.

e. The threats affected attribute (no. of threats degraded) is a measure of the ability of the EW system to affect individual threat types in a dense environment (currently operational and forecast threats validated by a threat group).

f. The threats defeated attribute (no. of threats defeated) measures the number of specific threat types defeated which are made inoperable by the EW system.

g. The cost attribute (dollars) is a measure of the life cycle costs of the EW system to include R and D, production, and maintenance of the expected life of the system. This total life cycle cost approach is applied with increasing frequency to DOD equipment acquisition programs.

h. The policy satisfaction attribute (direct effectiveness measure of directives, policies, and requirements satisfied) is a measure of the degree to which the EW retrofitted aircraft satisfies forecast government policies for the production decision point particularly of the G-2 group. Policy satisfaction includes concepts influencing candidate EW system political attractiveness at a point in time in the future (the

production decision point). This then requires one to forecast the political mood and subsequent government policy. Some of the elements which are included in this factor are: attitude toward U.S. defense posture by the administration and congress, need for foreign arms sales, treaty and alliance commitments, budgetary priorities and amounts of federal spending, military lobby, time to production of EW system, availability of manufacturers to meet the requirement for dual source suppliers of critical parts of a system, employment and unemployment effects on certain contractors, granting of subcontracts of a system for the sake of keeping a base of companies involved in defense oriented work, etc. None of the DMs interviewed were able to quantify all of the above elements, so an attempt at aggregation of the elements met with some success. The DMs were able to express the fact that as the time until production increased, the probability of accurately forecasting the political policies which would need to be satisfied decreased exponentially.

All DMs agreed after discussing that this factor needs inclusion even if it is very difficult to quantify accurately. This concensus among DMs and advisors for including policy satisfaction as one of the objectives justifies the use of a MCDT approach such as MOOT or MAUT. A normalized scale (limits 0.0 to 1.0) was used to score the alternatives with respect to policy satisfaction. The minimum score of 0.0 represents the case where it is estimated that an alternative will not satisfy any of the policies in effect at the production decision point. The maximum score of 1.0 represents the case where it is estimated that an alternative will satisfy all policies in effect at the production decision point. The DMs and advisors estimated policy satisfaction scores for the candidate systems. A maximum time for initiation of production was established at eight years so that the values could be obtained for various candidate systems with certain characteristics.

It is noted that no comprehensive set of attributes (as complete as the set just presented) is now used in the initial design stages of the current EWARD efforts. The DMs and advisors interviewed in this effort agreed that the above set of attributes covers the salient considerations

of an EW retrofit program, and that the goodness of a specific system could be adequately evaluated using these attributes.

The alternative policies that achieve the objectives with respect to an EW task are the selection and retrofit of EW equipment into the designated aircraft. A specific action is the selection and retrofit of a specific EW system with the designated number and type of the primary components (including associated airborne and ground equipment) as indicators of this activity (e.g. system α has 16 transmitters, 3 receivers, 1 processor and 4 expendables). Each separate system designated has characteristics measurable by the lower level attributes mentioned earlier (e.g. systems α could have weight β, life cycle cost Γ, degrade Ω threats, etc.). The attribute values and characteristics of each specific system in competition for selection can have deterministic and probabilistic values (e.g. R and D may not yet be completed and the weight, cost, and number of threats covered are not known with certainty, but utilizing data and estimates, distributions covering the stochastic elements were obtained).

A hypothetical deficiency designation and set of possible EW systems to ameliorate this vulnerability will now be described. A hypothetical situation is selected because the security classification pertaining to past and ongoing EW systems would prevent the publication of these results in unclassified texts thereby prohibiting a general usefulness which is the purpose of this research. While no information is compromised, the hypothetical situation characteristics are relevant to actual EW systems so that DMs and advisors could realistically take part in the EWARD effort (in the opinion of the DMs and advisors interviewed, the hypothetical situation mimicked reality to a high degree). The hypothetical situation was also used to produce a general solution procedure that may be situation specific, but not equipment specific (i.e., the modelling done was intended to be general in nature - applicable to any EW system of the present and near future - and not intended to concentrate only on modelling specific pieces of equipment for a single solution).

Assume the deficiency statements (MENS and SON) designated "X"

primary threats and N" secondary threats (specific active - radar, laser, etc., and passive - infra-red, electro-optical, etc.) which are sources of intelligence for enemy fire control systems and armaments (airborne interceptor, anti-aircraft artillery, surface-to-air missiles, etc.). A set of EW system components both developed and proposed to counter these N threats (N = N' + N") is available from governmental and industrial sources. For our hypothetical situation, the attribute limits for the possible system components are shown in Table 4.

A set of alternative proposed systems was assembled which are representative and typical of the spectrum of choices which confront the analysts and policy makers in the Conceptual Phase of EWARD. This list of alternative EW system configurations shown in Table 5 is hypothetically generated as a response of the RFP in the initial stage of the Conceptual Phase. These alternative configurations (with expected attribute levels and associated normal probability density functions were constructed from USAF supplied in-house and contractor empirical data, and supplemented by expert opinion when incomplete data was available (i.e., for the levels of the policy satisfaction attribute, the opinions of DMs and advisors from G-1, G-2, and G-3 were used to estimate levels of attainment for the alternatives and accompanying probability density functions, since little or no data was available for this task). The probability density functions (pdf) represent the risk of realizing a system (in the Production Phase) which has the attribute levels listed. Figure 8 shows an example of retrofit system weight and its accompanying probability density function for alternative 1. The alternatives are described in Table 5 by a dominant characteristic (i.e., high cost alternative, high electronic warfare performance alternative, etc.). The probability density functions for the alternatives in terms of standard deviation of their attribute levels is listed in Table 6. These density functions are utilized later in the impact assessment and decision making steps.

In order to establish a basis upon which to choose an appropriate MCDT approach, the attributes were investigated with respect to preferential independence. Selected DMs and advisors from each group were

Table 5

Alternative Retrofit System
Configurations and Characteristics

Alternative	Descriptive Phase
1 (a_1)	Compromise 1 (average)
2 (a_2)	High cost, high reliability
3 (a_3)	High electronic warfare performance
4 (a_4)	High aircraft performance
5 (a_5)	Low electronic warfare performance
6 (a_6)	Low aircraft performance
7 (a_7)	Low cost
8 (a_8)	Compromise 2 (average)

Attribute Values

Alt.	x_{1a}(kg) Weight	x_{1b}(m^3) Volume	x_{1c} (KVA) Power	x_2 ($Billion) Life Cycle Cost	x_{3a} Crew Performance	x_{3b} (threats) Threats Degraded	x_{3c} (threats) Threats Defeated	x_4 Policy Satisfaction
1	2.8×10^3	2.2	79.0	1.20	0.49	16.0	11.0	0.70
2	2.5×10^3	2.1	84.0	1.95	0.81	29.0	18.0	0.34
3	3.9×10^3	4.0	103.0	1.85	0.96	28.0	25.0	0.42
4	8.1×10^2	1.6	53.0	0.9	0.64	12.0	6.0	0.44
5	1.5×10^3	2.9	46.0	0.4	0.52	10.0	3.0	0.39
6	4.4×10^3	3.7	101.0	1.3	0.72	18.0	12.0	0.45
7	1.2×10^3	3.1	61.0	0.3	0.43	12.0	4.0	0.65
8	2.4×10^3	2.1	77.0	1.3	0.74	18.0	13.0	0.68

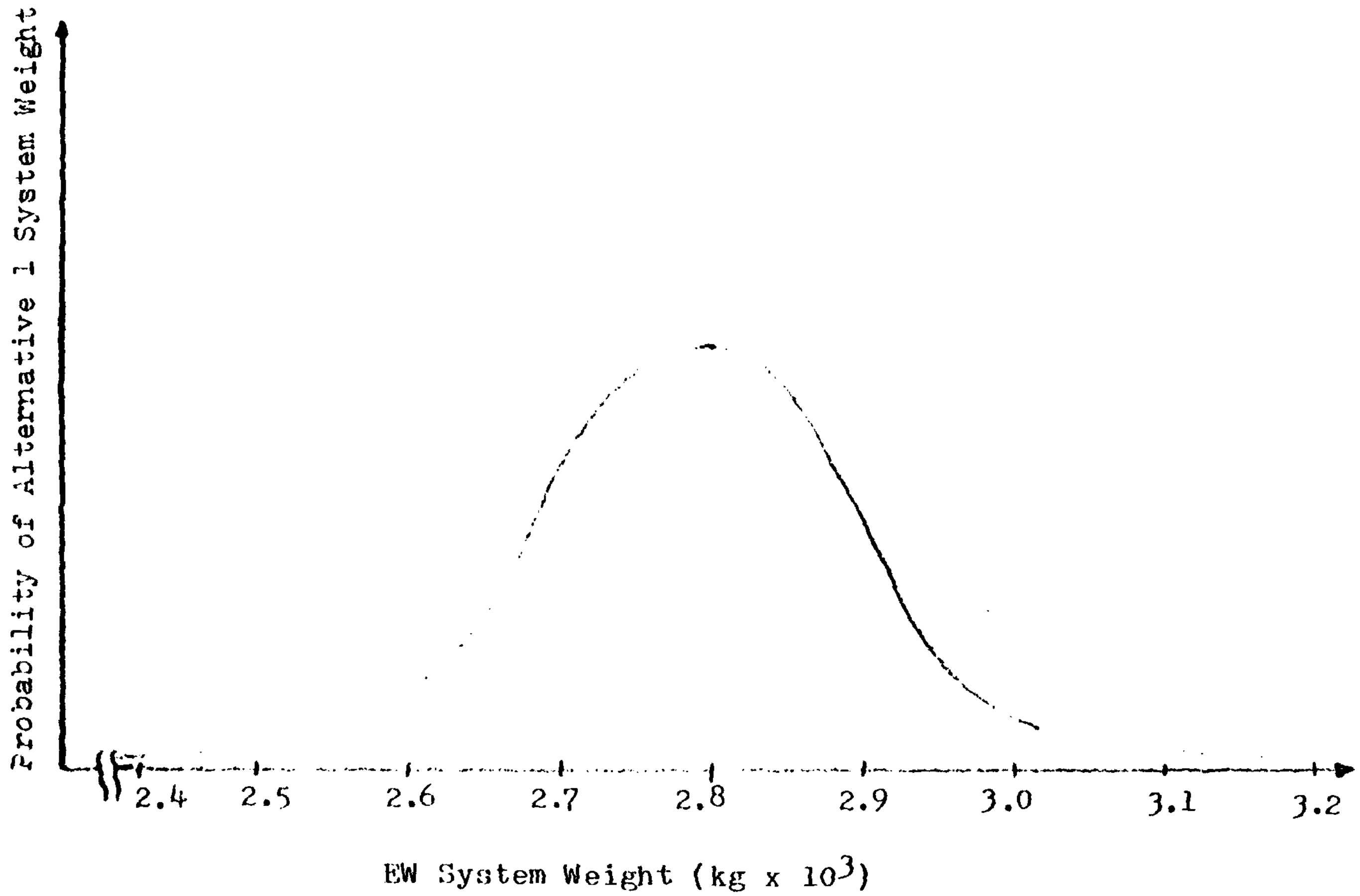


Figure 8. Probability Distribution Of EW System Weight For Alternative 1

Table 6

Alternative Configurations

Alternative	x_{1a}	x_{1b}	x_{1c}	σ_{1a}	σ_{1b}	σ_{1c}	$x_2(10^9)$	$\sigma_2(10^8)$	x_{3a}	x_{3b}	x_{3c}	σ_{3a}	σ_{3b}	σ_{3c}	x_4	σ_4
*1E	2800.	2.20	79.0				1.20		0.49	16.0	11.0				0.70	
*1D	2905.	2.68	84.1	62.5	0.24	2.55	1.29	0.45	0.37	12.0	9.0	.040	4.0	1.0	0.66	0.020
*1I	2695.	1.7	73.9				1.11		0.57	20.0	13.0				0.74	
2E	2500.	2.10	84.0				1.95		0.81	28.0	18.0				0.34	
2D	2580.	2.51	87.5	40.0	0.21	1.75	2.00	0.25	0.79	26.0	16.0	.010	1.0	1.0	0.26	0.040
2I	2420.	1.69	80.5				1.90		0.83	30.0	20.0				0.42	
3E	3900.	4.00	103.0				1.85		0.96	28.0	25.0				0.42	
3D	4055.	4.10	105.0	77.5	0.05	1.00	1.[illegible]5	0.50	0.95	27.0	24.0	.005	0.5	0.5	0.36	0.030
3I	3745.	3.90	101.0				1.75		0.97	29.0	26.0				0.48	
4E	810.	1.60	53.0				0.90		0.64	12.0	6.0				0.44	
4D	920.	1.50	57.2	55.0	0.05	2.10	1.10	1.00	0.54	7.0	3.0	.050	2.5	1.5	0.37	0.035
4I	500.	1.70	48.8				0.70		0.74	17.0	9.0				0.51	
5E	1500.	2.90	46.0				0.40		0.52	10.0	3.0				0.39	
5D	1615.	3.36	47.0	57.5	0.23	0.50	0.60	1.00	0.43	5.0	1.0	.045	2.5	1.0	0.33	0.030
5I	1385.	2.44	45.0				0.20		0.61	15.0	5.0				0.45	
6E	4400.	3.70	101.0				1.30		0.72	18.0	12.0				0.45	
6D	4485.	3.92	103.5	42.5	0.11	1.25	1.45	0.75	0.69	14.0	10.0	.015	2.0	1.0	0.38	0.035
6I	4315.	3.48	98.5				1.15		0.75	22.0	14.0				0.52	
7E	1200.	3.10	61.0				0.30		0.43	12.0	4.0				0.65	
7D	1295.	3.60	66.8	47.5	0.25	2.90	0.40	0.50	0.32	6.0	1.0	.055	3.0	1.5	0.70	0.025
7I	1105.	2.60	55.2				0.20		0.54	18.0	7.0				0.60	
8E	2400.	2.10	77.0				1.30		0.74	18.0	13.0				0.68	
8D	2508.	2.41	81.4	54.0	0.16	2.20	1.42	0.60	0.68	15.0	12.0	.030	3.0	0.5	0.62	0.030
8I	2392.	1.79	72.6				1.18		0.80	21.0	14.0				0.74	

*E = Expected Configuration

*D = Degraded Configuration

*I = Improved Configuration

Attribute and Accompanying Standard Deviation Values

interviewed to examine how these gerents trade-off levels of the attributes. Using standard assessment techniques, second order preference independence was explored (Keeney and Raiffa, 1976; Keefer, 1978; Keeney, 1974). The major attributes ($\underline{X}_1$, X_2, $\underline{X}_3$, X_4) were found to be preferentially independent (PI) and the components of $\underline{X}_1$ and $\underline{X}_3$ were also found to be PI (an example of the PI assessment for Group 3 is shown in Figure 9). The set $(\overline{X}_i, \overline{X}_j)$ is preferentially independent of $X_{\overline{ij}}$ (set levels of all other attributes except X_i and X_j) if preferences for consequences differing only in the value of X_i and X_j do not depend on the fixed value of $X_{\overline{ij}}$.

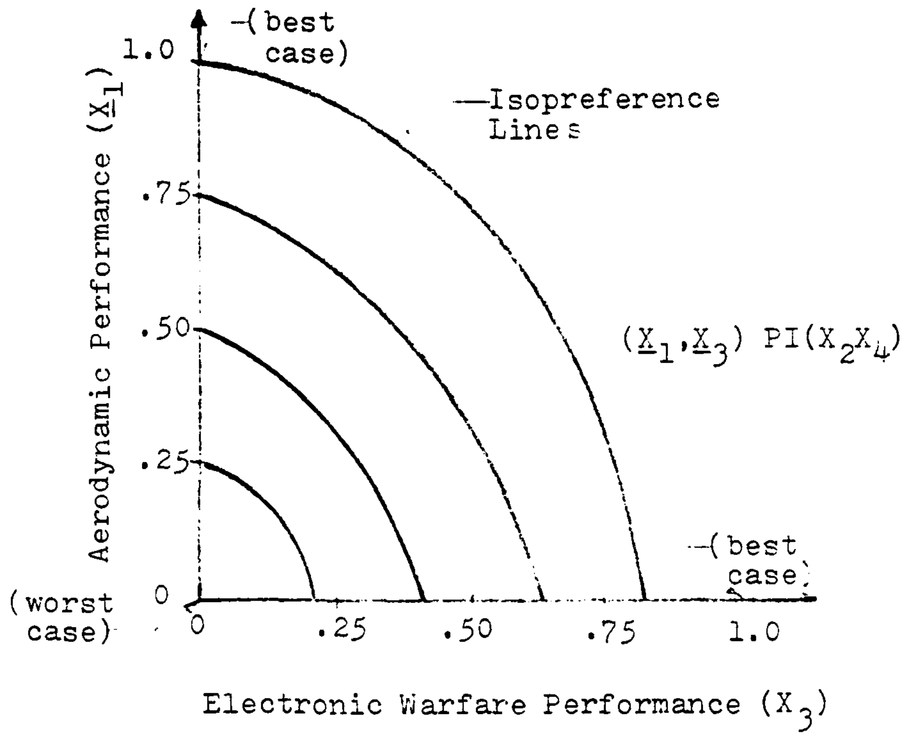
An appropriate point for the remainder of the EWARD to take place is between the Conceptual Phase authorization and Program Management Plan preparation. Its results should be used to aid in judging the contractor competition or sole source performance, and also used as inputs to the final RFP after contractor selection before production. The information needed to perform the EWARD as presented in this chapter is present in the Conceptual Phase.

Now that a basic pre-analysis has been conducted for the EWARD situation presented, the remaining steps of modelling and system analysis, optimization and decision making are accomplished in Section 4 using a combined MOOT/MAUT approach.

4. MOOT/MAUT Approach to EWARD

Factors in EWARD such as the lack of an adequate scalar performance measure, the high level of complexity, and the difficulties cited which hamper an efficient acquisition process, make this situation a likely candidate for a comprehensive multiple criteria approach like the combined MOOT/MAUT process applied through the systems engineering methodology.

The EWARD situation contains certain characteristics such as a set of attributes which the DMs and advisors have established as preferentially independent and an indication by the DMs and advisors that they would prefer to reduce the number of alternatives before comparing and ranking the remaining alternatives. The DMs and advisors traditionally



($\underline{X}_1$ and $\underline{X}_3$ were consistently traded off according to the curves above for various joint levels of X_2 and X_4).

Figure 9. Preference Independence Of Attributes

attempt to eliminate some alternatives before examining the remaining in more depth, and were comfortable working in this mode. These characteristics in EWARD suggest that the combined MOOT/MAUT process is appropriate for modelling and resolving this decision situation. The MOOT/MAUT process as applied to EWARD will follow the basic algorithm outlined in Figure 2 (Chapter 4) with some minor variation due to the fact that there are groups of DMs involved. The main steps are: a) conduct a pre-analysis phase, b) eliminate the inferior alternatives through an optimization of process and an elimination by aspects exercise, c) elicit the preference structure of the DMs to subsequently develop a scalar choice function which ranks the remaining alternatives to identify the optimal policy, d) conduct a sensitivity analysis and validation exercise, e) prepare an action plan. The pre-analysis of Section 3 will be used as the basis for modelling, optimization, and decision making.

In response to the development of a MENS, retrofit system configurations from the contractors become available which represent the alternative actions. All of the proposed alternative systems are feasible alternatives, and no preliminary scanning is required in EWARD at this point in the effort to eliminate the unacceptable alternatives. Because a large amount of engineering design is required for interfacing a complete retrofit system, information is available at this point (Conceptual Phase) which basically establishes the impacts of each alternative action (implementing an alternative retrofit system). Therefore, no tuning or refining of the alternative systems is required in the Conceptual Phase unless none of the alternative actions are viable solutions to the deficiencies cited in the MENS. The criteria established in the pre-analysis phase was used as the basis for developing the attributes (Table 1) used in this effort.

The identification of alternatives which are superior with respect to the attributes developed in the pre-analysis phase is undertaken with the formation of the MDSS. In order to utilize the data concerning the attributes (which include risk information), the technique of stochastic dominance was used to indicate which of the alternatives was dominated. The eight alternatives were compared to each other with

respect to the stochastic dominance of attribute values to see if any alternatives were dominated with respect to all attributes. The cumulative probability distribution (CDF) for attribute values for each of the alternatives with respect to system weight (x_{1a}) is shown in Figure 10. When the area under the CDF for an alternative (A_i) is less than the area under the CDF of another alternative (A_j) (for a specified attribute), A_i is said to dominate A_j in the sense of stochastic dominance for that attribute. A_i is said to dominate A_j if stochastic dominance is present for all attributes for A_i with respect to A_j. Stochastic dominance relationships were established for all alternatives on a pairwise basis for all attributes with the results that only one alternative system (alternative 6) is dominated. The large NDSS was expected since there are a number of conflicting objectives. In the interest of reducing the number of alternatives in the NDSS further, to bring the NDSS membership to a number acceptable by the DMs, an elimination by aspects exercise was conducted. Each group was asked to come up with a set of realistic (from their specific vantage point) minimum acceptable attainment levels that the alternatives would have to pass to be considered in the nondominated solution set. These levels are shown for all three groups in Table 7. In this elimination by aspects exercise, the analyst must ensure that all attainment levels of the attributes which are exceeded by an alternative reflect an essential shortcoming in the EWARD. This elimination by aspects eliminated three alternatives (alternatives 2, 3, and 5) each of whose attribute values violated many minimum attainment levels of the stakeholder groups. There were then four alternatives comprising the NDSS (alternatives 1, 4, 7, and 8).

Now in order to select the best alternative system from the reduced NDSS, and to facilitate operating with risk in the decision process, a MAUT technique was utilized for the ranking of alternative systems/decision making phase. The characteristics of the situation such as an available set of attributes, the availability of the directly assessed preference structure of the DMs and advisors, and the availability of probabilistic information concerning the attribute values for the alternatives, suggest the use of Multiattribute Decision Analysis as

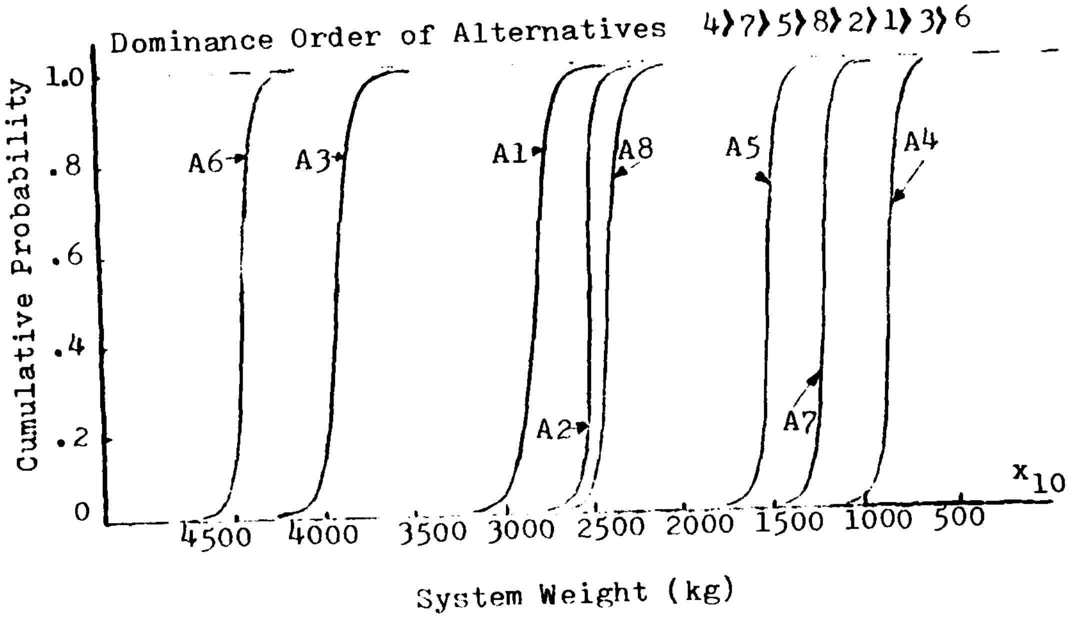


Figure 10. Stochastic Domination

Table 7

Required Attainment Levels

Attributes	Group 1	Group 2	Group 3
X_{1a}(kg)	$\leq$ 3000.00	$\leq$ 4500.00	$\leq$ 3500.00
X_{1b}(m^3)	$\leq$ 3.20	$\leq$ 4.10	$\leq$ 3.60
X_{1c}(KVA)	$\leq$ 90.00	$\leq$ 105.00	$\leq$ 90.00
X_2 (\$x10^9)	$\leq$ 1.50	$\leq$ 1.40	$\leq$ 1.70
X_{3a} *	$\geq$ 0.50	$\geq$ 0.00	$\geq$ 0.45
X_{3b}(threats)	$\geq$ 16.00	$\geq$ 10.00	$\geq$ 9.00
X_{3c}(threats)	$\geq$ 10.00	$\geq$ 2.00	$\geq$ 5.00
X_4 *	$\geq$ 0.33	$\geq$ 0.50	$\geq$ 0.40

* Normalized Scale

$\leq$ Maximum Attainment Level

$\geq$ Minimum Attainment Level

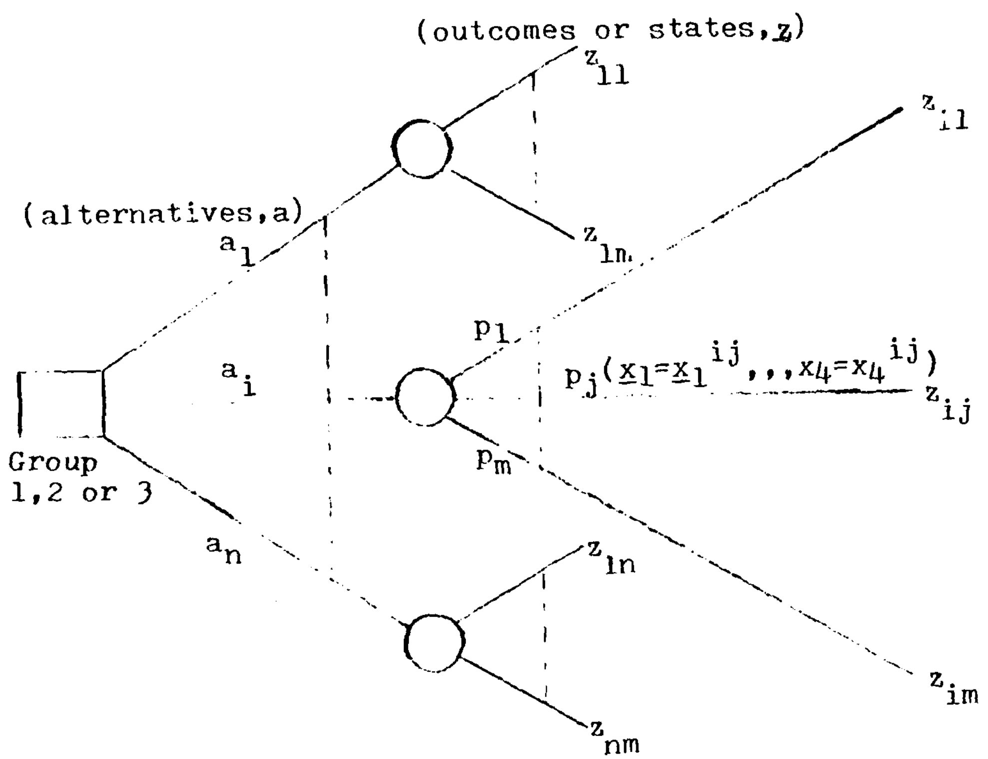
the MAUT technique.

Multiattribute Decision Analysis (MADA) has been a very popular technique for use in decision situations with multiple attributes. To use this requires the identification of subjective probability for uncertain consequences, and elicitation of DM's preferences over these attributes expressible in cardinal utility functions (Keeney and Raiffa, 1976; Keeney and Wood, 1977; Brown, Kahr, and Peterson, 1974; Barclay, et al., 1977; Huber, 1977; Sage, 1977; Keeney and Kirkwood, 1977). The added complication of multiple DMs and advisors at various levels requires the consideration of group amalgamation of preferences (Banker and Gupta, 1978; Keeney and Kirkwood, 1975; Nakayama, et al., 1979).

The basic decision model is illustrated on Figure 11. The decision space a A is made up of individual actions of the reduced NDSS

$a_i \equiv$ incorporating set i (alternative i) of equipment as the retrofit system, i=1,2,,,,n

i.e., set i includes a combination of EW equipment with associated attribute values for X_i, i = 1,2,3,4 as discussed in Section 3. For example, selecting a_1 (implementing alternative 1) causes the retrofit of a system with components which collectively give the attribute levels z_{11}; i.e., $\underline{x}_1^{z_{11}}, x_2^{z_{11}}, \underline{x}_3^{z_{11}}, x_4^{z_{11}}$) with probability p_1 where p_1 is the combined probability of $\underline{x}_1 = \underline{x}_1^{z_{11}}, x_2 = x_2^{z_{11}}, \underline{x}_3 = \underline{x}_3^{z_{11}}, x_4 = x_4^{z_{11}}$. The probability of the system configuration under alternative 1 being at other collective attribute levels or output states is p_i where i = 1,2,,,m and $\sum_{i=1}^{m} p_i = 1$. These probabilities express the likelihood of the retrofit system giving the estimated values of the attributes in actual operation on the aircraft. In order to incorporate the risk of system implementation into the MADA format, output configurations other than the expected configuration were constructed from the data and combined with probabilities of realization from Monte Carlo simulations. In order to keep the problem in a framework that current EWARD stakeholders are familiar with, only three resulting configurations were used for each alternative configuration. The characteristics of these three configurations (a degraded, expected, and improved version) of each alternative system is shown in Table 6. Because of the gaussian density function



z_{ij} = Configuration j of Alternative System i

$u(z_{ij}) = f(u_1(\underline{x}_1^{ij}),,,u_4(x_4^{ij}))$

$\sum_{j=1}^{m} p_j = 1$

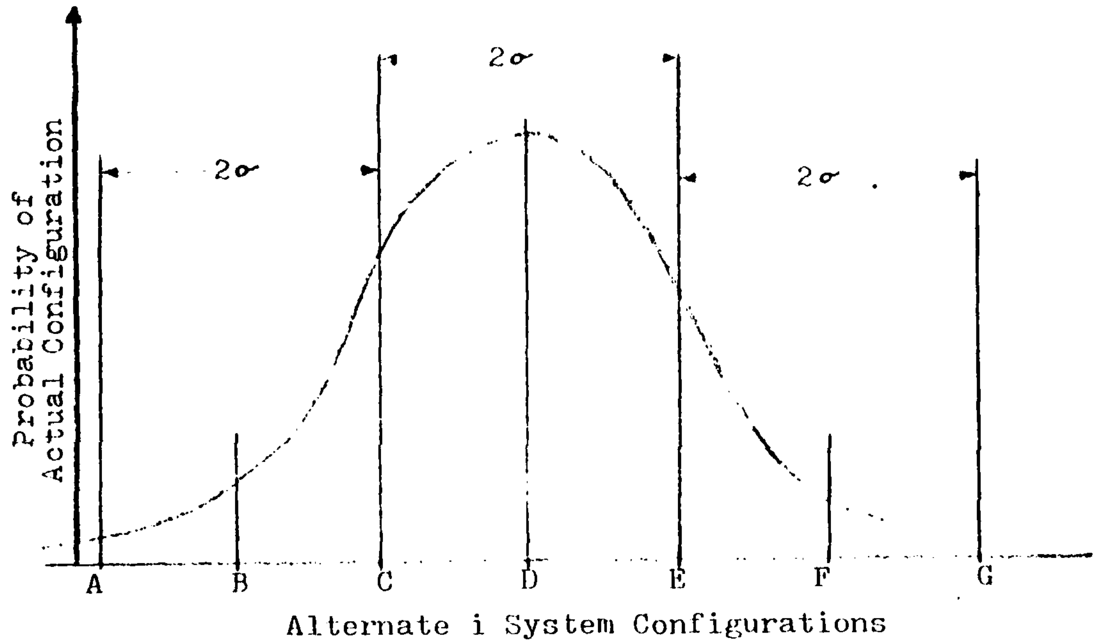
Initial ranking of Alternatives (decisions) is accomplished by averaging out the single stage decision tree: i.e. $\underset{a}{\text{Max}}\ E(u(z,a)) = \underset{a_i}{\text{Max}} (\sum_j p_j(a_i)u(z_{ij}(a_i)))$

Figure 11. Multiattribute Decision Analysis Model

forms for all candidate systems, three standard probabilities were used for the three approximate outputs of each candidate system (p = .68 for the expected configuration state, p = .16 for the improved configuration state, and p = .16 for the degraded configuration state). Figure 12 illustrates the rationale behind this approximation. This discretization of the probabilistic data means that m = 3 and n = 8 for the EWARD formulation of MADA posed in Figure 11.

This MADA technique was modelled for application chronologically in the Conceptual Phase of EWARD so as to be able to incorporate available information from all groups at an early phase, and to specify the retrofit configuration and evaluate the resulting system. Care was taken to separate the subjective probability encoding and utility assessments so as to prevent any interaction effects such as bias and double weighting of certain events and outputs.

The first items addressed were the functional relationships between attributes and the form of the multiattribute utility function which would measure an aggregated felicity for each alternative retrofit system. Since preferential independence among attributes was established in the pre-analysis phase, first order utility independence (UI) was next examined. X_i is UI of $X_{\bar{I}}$ (all attributes other than X_i) if preferences for risky choices (lotteries) over X_i with the value of $X_{\bar{I}}$ held fixed do not depend on the fixed value of $X_{\bar{I}}$. Using standard MAUT assessment techniques (Keeney and Raiffa, 1976), it was found that $\underline{X}_1$ was utility independent and because $(\underline{X}_1, X_i)$ is PI, i = 2,3,4, using Theorem 6.2 (Keeney and Raiffa, 1976) it can be concluded that the major attributes are mutually UI (MUI). The components of each of $\underline{X}_1$ and $\underline{X}_3$ were also found to be MUI. Next, the attributes were examined with respect to Fishburn Marginality (Fishburn, 1967; Winterfeldt and Fischer, 1973) as shown in Figure 13. It was soon pointed out by the interviewees that additive independence did not hold. Therefore invoking the fact that the attributes are MUI and Theorem 6.1 of Keeney and Raiffa (1976), it is concluded that a multiplicative form of multiattribute utility function is appropriate for this application. The multiplicative utility function has the general form



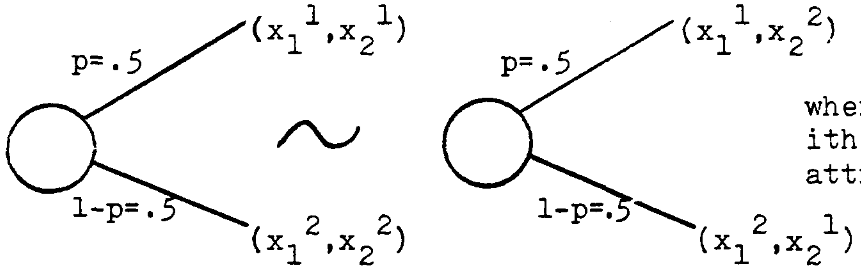
*A,B,C,D,E,F,G = Alternative i System Configurations

*D ≡ Expected Configuration; Probability ≃ .68 that the actual configuration is between configurations C and E

*F ≡ Improved Configuration; Probability ≃ .16 that the actual configuration is between configurations E and G

*B ≡ Degraded Configuration; Probability ≃ .16 that the actual configuration is between configurations A and C

Figure 12. Discrete Approximation Of Configuration Incorporated Risk



where x_j^i is the ith level of attribute j

The additive utility function requires the individual utility function $(u_i(x_i))$ to be additive independent. Additive independence occurs when preferences over lotteries on outcomes (described in terms of the attributes) depend only on marginal probability density functions of the attributes, $f(x_i)$, and not on the joint density functions of the attributes $f(\underline{x}) = f(x_1, x_2, , , x_n)$. That is, the two lotteries shown below must be equally preferred. This condition must hold for all combinations of the attributes $(x_1, x_2, , , x_n)$; i.e. if $n = 4$, this condition must hold for all possible subsets of (x_1, x_2, x_3, x_4).

Figure 13. Check For Marginal Equivalence (Marginality)

$$1 + Ku = \prod_i (1 + KK_i u_i) \quad (1)$$

where u is the combined utility function, u_i is the i^{th} constituent utility function, K_i is a scaling constant for the i^{th} utility function, and K is the scaling constant for the combined utility function. Specifically, the utility functions for attribute $\underline{X}_1$ and $\underline{X}_3$ respectively are

$$1 + k_1 u_1 = (1 + k_1 k_{1a} u_{1a}(x_{1a}))(1 + k_1 k_{1b} u_{1b}(x_{1b}))(1 + k_1 k_{1c} u_{1c}(x_{1c})) \quad (2)$$

and

$$1 + k_3 u_3 = (1 + k_3 k_{3a} u_{3a}(x_{3a}))(1 + k_3 k_{3b} u_{3b}(x_{3b}))(1 + k_3 k_{3c} u_{3c}(x_{3c})) \quad (3)$$

where u_{mn} is the lower level attribute utility function, u_j is the higher level utility function, and the k's are scaling constants. The aggregate utility function for each group is

$$1 + KU_i = (1 + KK_1 u_1)(1 + KK_2 u_2)(1 + KK_3 u_3)(1 + KK_4 u_4) \quad (4)$$

where U_i is the combined utility function for each group, i=1,2,3, and the K's are scaling constants. The multiplicative form adds complexity of analysis compared to the additive form, but also supplies the required non-compensatory inter-attribute relations necessary in this problem (Appendix A).

The utility independence condition (which was verified for all attributes) allows the assessment of eight single dimension utility functions. The utility function for each attribute was needed to incorporate the DM's and advisor's preferences and attitude toward risk. Because of the many gerents at various levels in EWARD, a refinement form of social choice function was used to bring the individual utilities of the DMs and advisors together. Using a modified form of the multiple independent entity/MCDM process of Banker and Gupta (1978) intended for decentralized DMs, the utility functions were extracted for each attribute from the lower level advisors and DMs in each group (G-1, G-2, and G-3). These utility functions were then presented to intermediate and eventually high level DMs in each group for refinement. These refined utility functions were then shown to each level of DM until concensus was reached on the final form for each group (G-1, G-2, and G-3). This way of incorporating the group utility into a single

function seemed to work well in this application because it followed the basic bureaucratic "chain of command" hierarchial decision making process of the government (lower level advisors counsel higher level DMs and higher level DMs feedback policy which focuses this advice thereby causing rapid convergence to a single social choice function or opinion). The refined group utility functions (utilizing the data from the refinement interviews was curve fit using the regression analysis subroutines NLWOOD; Meeter, et al., 1977; and computer programs like MANACON, Schlaifer, 1971) for the individual attributes are listed in Table 8. An example of the group utility curves for attribute X_{3b} (number of threats degraded) is shown in Figure 14. The majority of the utility curves exhibit a preference structure which is risk averse as illustrated Figure 14. The risk averse tendencies by the DMs and advisors were expected in a project which involves government officials (Levy , 1974).

Since concensus established the multiplicative form of combined multiattribute utility function, the task of evaluating the scaling constants for this form were approached next. Using suggested assessment techniques (Keeney and Raiffa, 1976) with the interviewees, the group utility functions (Table 8) and the attribute ranges (Table 4), the scaling constants were evaluated for each group and the results are shown in Table 9 for the multiplicative utility functions. As one examines Table 9, each group's relative strengths of the preferences for the attributes become evident. For instance, the values of the combined utility function scaling coefficients show that Group 2 prefers a high level policy satisfaction (K_4) and low cost (K_2) over performance (K_1 and (K_3) while Group 3 values aerodynamic (K_1) and electronic warfare performance (K_3) over political (K_4) and financial (K_2) considerations.

When the sum of the constituent utility function coefficients is not equal to 1.0, then a dependency among attributes, which is signified by the multiplicative form, is verified. When the sum of the constituent function scaling constants is greater than one, the combined utility constants (K, k_1 or k_3) is between minus one and zero (at minus one, all constituent scaling constants must be equal to one). This condition signifies a case where a group's strengths of the preferences for

Table 8

Group Utility Functions For Each Attribute

	Group 1	Group 2	Group 3
$u_{1a}(x_{1a})$	$1.52-0.44\exp(3.1E-4\,x_{1a})$; $x_{1a} \leq 3500$ $-0.97+2.4\exp(-2.0E-4x_{1a})$; $x_{1a} \geq 3500$	$1.13-2.5E-4\,x_{1a}$	$1.08-0.06\exp(6.5E-4x_{1a})$
$u_{1b}(x_{1b})$	$1.26-0.10\exp(0.61x_{1b})$	$1.65-0.39\exp(0.35x_{1b})$	$1.41-0.19\exp(0.49x_{1b})$
$u_{1c}(x_{1c})$	$1.37-0.15\exp(0.02\,x_{1c})$	$1.28-0.09\exp(0.02\,x_{1c})$	$1.19-0.05\exp(0.03x_{1c})$
$u_2(x_2)$	$1.13-0.11\cdot\exp(1.15\,E-9\,x_2)$	$1.01-0.01\exp(2.1\,E-9x_2)$	$1.18-0.16\exp(1.0\,E-9\,x_2)$
$u_{3a}(x_{3a})$	$0.99(x_{3a})^{\frac{1}{2}}$	$-1.01+1.0\exp(0.7(x_{3a})^{1.1})$	$-2.97-0.35\exp(-5.1x_{3a})$ $+3.32\exp(0.18x_{3a})$
$u_{3b}(x_{3b})$	$4.92-5.15\exp(-0.01\,x_{3b})$	$2.53-2.80\exp(-0.02x_{3b})$	$1.18-1.72\exp(-0.08x_{3b})$
$u_{3c}(x_{3c})$	$1.52-1.53\exp(-0.04\,x_{3c})$	$1.59-1.59\exp(-0.02\,x_{3c})$; $x_{3c} \leq 20$ $-0.52+0.60\exp(0.03(x_{3c})$; $x_{3c} \geq 20$	$1.01-1.01\exp(-0.13x_{3c})$
$u_4(x_4)$	$1.0x_4$	$1.01-1.01\exp(-5.1x_4)$	$-1.0+1.01\exp(0.69x_4)$

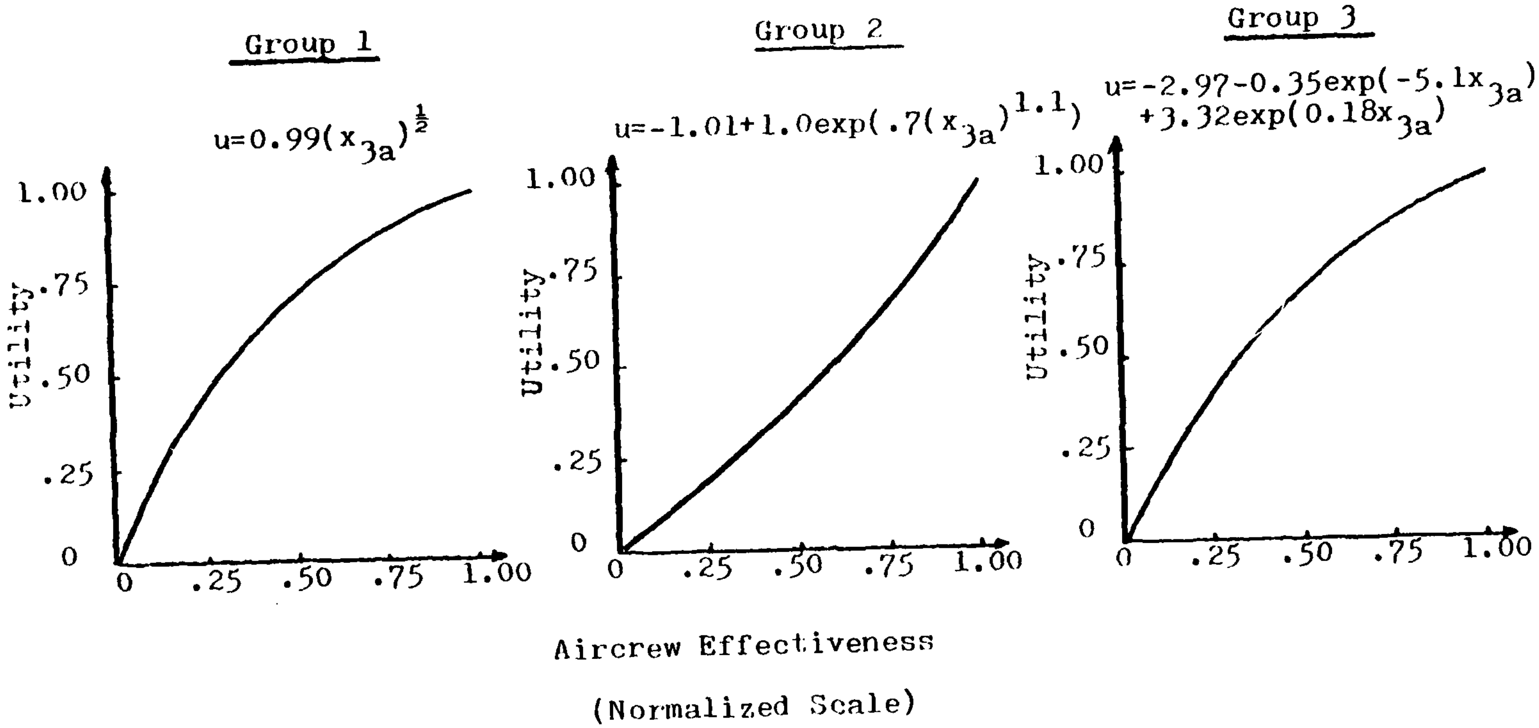


Figure 14. Group Utility Functions For Attribute X_{3a}

Table 9. Utility Functions For The Stakeholders

Aggregated Utility Functions

$$1+KU_i = (1+KK_1u_1)(1+KK_2u_2)(1+KK_3u_3)(1+KK_4u_4)$$

where U_i = combined group utility function, i = 1,2,3

u_j = higher level attribute constituent utility function
j = 1,2,3,4

Scaling Coefficients	Group 1	Group 2	Group 3
K	-0.76	-0.97	-0.98
K_1	0.51	0.30	0.69
K_2	0.33	0.75	0.46
K_3	0.49	0.32	0.92
K_4	0.21	0.80	0.45

Attribute Utility Functions

$$1+k_1u_1 = (1+k_1k_{1a}u_{1a}(x_{1a}))(1+k_1k_{1b}u_{1b}(x_{1b}))(1+k_1k_{1c}u_{1c}(x_{1c}))$$

$$u_2 = u_2(x_2)$$

$$1+k_3u_3 = (1+k_3k_{3a}u_{3a}(x_{3a}))(1+k_3k_{3b}u_{3b}(x_{3b}))(1+k_3k_{3c}(x_{3c}))$$

$$u_4 = u_4(x_4)$$

where u_{mn} = lower level attribute utility function;
m = 1,3   n = a,b,c

Scaling Coefficients	Group 1	Group 2	Group 3
k_1	-0.60	-0.29	0.14
k_{1a}	0.48	0.40	0.38
k_{1b}	0.59	0.37	0.26
k_{1c}	0.24	0.35	0.31
k_3	-0.40	5.04	-0.97
k_{3a}	0.51	0.12	0.42
k_{3b}	0.29	0.25	0.48
k_{3c}	0.38	0.13	0.95

attributes are medium to high (e.g. k_3 = -.97 for group 3). When the sum of the constituent scaling constants is less than one, then the combined utility constant is greater than zero. This condition signifies a case where a group exhibits medium to weak preferences for the attributes of interest. As the magnitude of a combined utility constant increases, the preferences tend to weaken in strength (eq. k_3 = 5.04 for group 2).

Each group's (G-1, G-2, G-3) utility for the set of alternatives described in Table 5 and Table 6 were calculated using the multiplicative form and scaling constants listed in Table 9 . The constituent utilities for the expected values of the candidate systems for all three groups are listed in Table 10. The combined group felicity for each alternative EW retrofit system in the reduced NDSS (expected version, improved version, and degraded version) is shown in Table 11a. The ranking of alternatives step was next accomplished by maximizing the expected individual group utilities according to Figure 11. The resulting group utilities for each alternative were generated by averaging out the single stage MADA formulation of Figure 11 for each alternative using the equation

$$E(u(z,a_i)) = \sum_j p_j(a_i)u\ (z_{ij}(a_i)) \qquad (5)$$

where $p_j(a_i)$ is the probability of outcome z_{ij}, and $u(z_{ij}(a_i))$ is the multiattribute utility of outcome z_{ij}, and $E(u(z,a_i))$ represents the expected utility or score for alternative a_i. The utility data of Table 11a and the probability data discussed earlier and illustrated in Figure 12 were substituted into this expected utility formulation. Table 11b shows the output of these calculations and the resulting preference rankings for the individual groups with respect to the alternatives. The closeness of the utility scores for each group is explained by the fact that even though utilities were elicited for the full range of each attribute, the alternatives in the NDSS had attribute values that were in a much narrower range.

The rankings of Figure 15, show that the G-1 (operations, intelligence) prefer the alternatives which aid aircraft performance and are

Table 10

Utility Values For The Expected Alternative Configurations

Alternatives	$u^*_{1a_1}$	$u^*_{1a_2}$	$u^*_{1a_3}$	$u^*_{1b_1}$	$u^*_{1b_2}$	$u^*_{1b_3}$	$u^*_{1c_1}$	$u^*_{1c_2}$	$u^*_{1c_3}$	$u^{**}_{A_1}$	$u^{**}_{A_2}$	$u^{**}_{A_3}$	u'_{2_1}	u'_{2_2}	u'_{2_3}
1	.47	.43	.72	.87	.81	.85	.58	.61	.64	.75	.65	.70	.68	.82	.65
4	.95	.93	.98	.98	.97	.99	.91	.93	.94	.98	.97	.86	.81	.91	.78
7	.88	.83	.95	.58	.49	.54	.83	.85	.87	.80	.76	.77	.97	.98	.96
8	.59	.53	.80	.89	.84	.87	.60	.63	.66	.80	.65	.70	.62	.78	.59

Alternatives	$u^*_{3a_1}$	$u^*_{3a_2}$	$u^*_{3a_3}$	$u^*_{3b_1}$	$u^*_{3b_2}$	$u^*_{3b_3}$	$u^*_{3c_1}$	$u^*_{3c_2}$	$u^*_{3c_3}$	$u^*_{C_1}$	$u^*_{C_2}$	$u^*_{C_3}$	u'_{4_1}	u'_{4_2}	u'_{4_3}
1	.69	.38	.63	.47	.52	.65	.47	.36	.83	.60	.28	.90	.70	.98	.62
4	.79	.53	.75	.28	.32	.47	.27	.20	.54	.50	.17	.72	.44	.90	.37
7	.65	.32	.58	.28	.32	.47	.18	.14	.39	.41	.12	.55	.65	.97	.57
8	.87	.65	.82	.54	.57	.73	.54	.41	.83	.73	.43	.95	.68	.97	.60

* u_{ij_k} = utility of component j (j = a,b,c) of attribute $\underline{X}_i$ (i = 1,3) for Group k (k = 1,2,3)

** u_{m_n} = combined utility of attribute X_m (m = A,C,A for $\underline{X}_1$ and C for $\underline{X}_3$)for Group n (n = 1,2,3)

' u_{s_t} = utility of attribute X_s (s = 2,4) for Group 5 (t = 1,2,3)

Table 11a

Combined Utility For The Various Configurations Of Alternatives

Alternatives	u_{1E}	u_{2E}	u_{3E}	u_{1D}	u_{2D}	u_{3D}	u_{1I}	u_{2I}	u_{3I}
1	0.78	0.96	0.96	0.76	0.94	0.94	0.80	0.98	0.97
4	0.83	0.97	0.93	0.82	0.95	0.89	0.86	0.98	0.94
7	0.78	0.97	0.90	0.77	0.90	0.89	0.81	0.94	0.90
8	0.82	0.95	0.97	0.80	0.93	0.94	0.84	0.97	0.98

u_{ij} = utility for configuration j (j = E(Expected, I(Improved), D(Degraded)) of Group i (i = 1,2,3)

Table 11 b

Group Preference Rankings Of Alternatives

Alternatives	u_1	Ranking	u_2	Ranking	u_3	Ranking
1	0.781	4	0.965	2	0.958	2
4	0.833	1	0.968	1	0.923	3
7	0.784	3	0.959	3	0.906	4
8	0.821	2	0.951	4	0.965	1

u = total utility for given alternative of Group k (k=1,2,3)

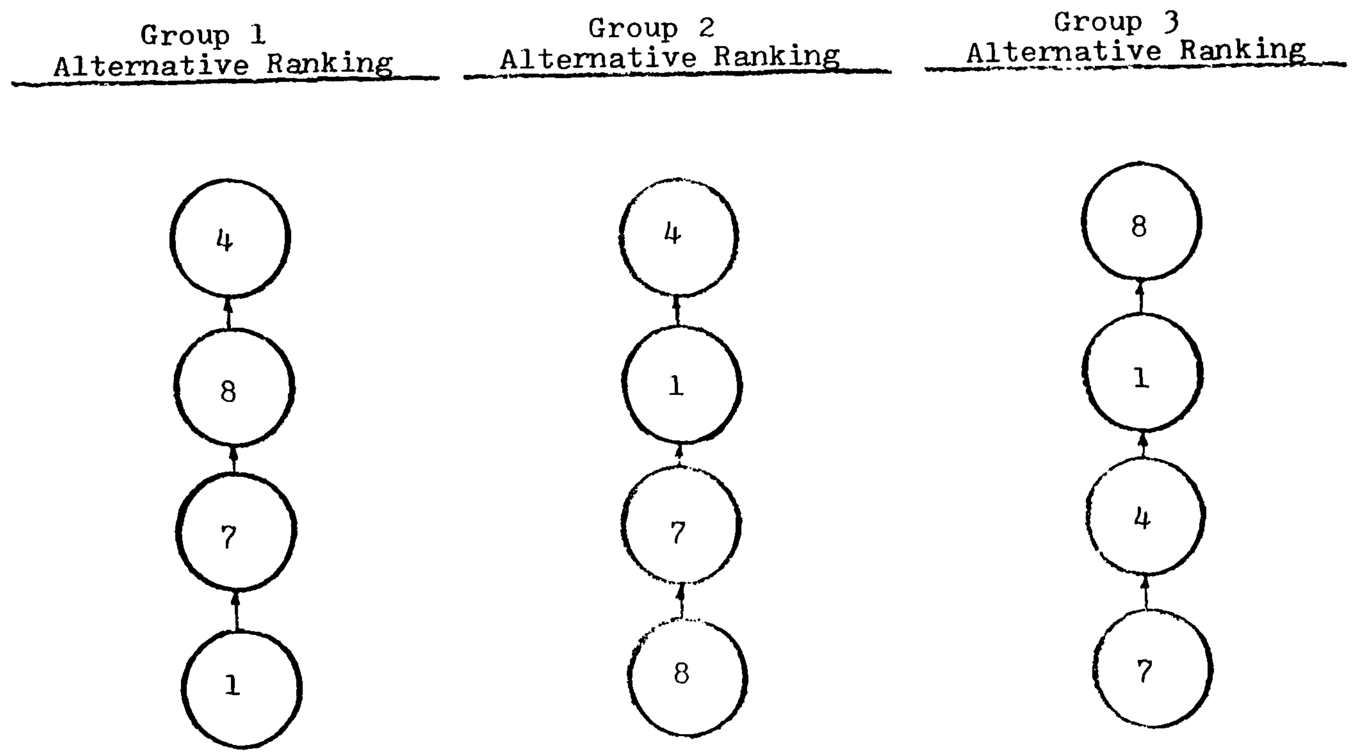


Figure 15. Individual Group Rankings Of The Candidate Systems

average or compromise systems. This was expected since many of the participants in G-1 have air crew experience (pilots, navigators, weapons operators, etc.). G-2 preferred the compromise and low cost alternatives. This also was expected since budgetary constraints and political agreements are driving forces for the government policy makers. It is noted that there is a correlation in the upper levels of alternative choices of G-1 and G-2 which is possibly precipitated by the close interaction between the DMs and advisors in G-1 and G-2 discussed in Section 2. G-3 (program managers, contractors, analysts, etc.) preferred the high EW and aircraft performance, high cost, and compromise alternatives. These choices were anticipated since engineers, contractors, and analysts in G-3 are basically performance oriented and consider cost secondarily. Alternatives 4 (high aircraft performance) and 8 (best compromise) were highly preferred by all groups. Because no alternative was ranked as most preferred groups, and to continue with the theme of a general application, the final modelling step was the combination of the three separate group (G-1, G-2, and G-3) multiattribute utility functions into a single social choice function so that final ranking can be performed for the EWARD presented. Because it can be seen from Table 11b, and Figure 15 that there are conflicting ranking for alternatives, a means was required to obtain combined group concensus. Because there are three groups and more than three alternatives, there was no way of obtaining concensus (through popular techniques like simple majority rule, simple additive weighting, etc.) without violating Arrow's axioms in the impossibility theorem (Arrow, 1963). This is because interpersonal comparison of utilities is required. Nevertheless, concensus is required, and two techniques which can aggregate the group utilities are the cardinal social welfare function discussed by Keeney and Kirkwood (1975) and the Extended Contributive Rule method (ECR) discussed by Nakayama, et al., (1979).

The Extended Contributive Rule method (ECR) was used to aggregate individual groups' preferences into a single choice function (Nakayama, et al., 1979). ECR amalgamates the preferences of the DMs in a way that incorporates directly the degree of confidence of all the individual

groups in their own preferences and in other groups' preferences, and the intensity of each preference. ECR was particularly applicable to EWARD since it considers preferences between two alternatives at a time and therefore gives an output which is readily transformed into an ISM digraph (Warfield, 1976) for display purposes. The ECR method of combining group preferences was chosen over the simple linear additive and multiplicative forms of combining group utility in this application because of two reasons:

1. ECR specifies a way of making interpersonal comparison of utility (each group ranks the importance of the opinion of all groups so that weights can be established for each group's utility). This concept of intergroup weighing makes intuitive sense in a governmental setting where the various groups realize their relative position with respect to the political structure which must eventually authorize funding for the EWARD production.

2. ECR method allows a preference threshold to be incorporated into the ranking of alternatives step. This thresholding feature affords the DM the opportunity to not only establish specific preference relations between alternatives, but also reveal the strengths of these preference relations. These pairwise preference relations can then be easily displayed on a digraph.

The ECR algorithm is now described. For a set of alternatives $A = (A_1,,,,,,,A_n)$, and group utility functions $u_i = (u_1, u_2, u_3)$, let w_{yz} be the weight which group y imposes on the utility of group z.

That is, w_{yz} represents group y's perception of the fraction of the whole opinion that group z influences. We define the quantity

$$\bar{A} = \sum_{i=1}^{3} w_i \cdot \Delta u^i_{jk} + \alpha\left(\sum_{i=1}^{3} w_i \cdot \mathrm{Min}(0, \Delta u^i_{jk}) - \beta\right) \qquad (6)$$

where $w_i = (\sum_{y=1}^{3} w_{yi})/3, \alpha \geq 0, \beta \geq 0$, and $\Delta u^i_{jk} = u_i(A_j) - u_i(A_k)$.

For two alternatives $A_j, A_k \in A$; A_j is preferred to A_k ($A_j \succ_\alpha A_k$), at an opinion level α if and only if $\bar{A} > 0$. All possible combinations of alternatives taken two at a time are categorized with respect to a

preference existing or not (binary relation). The parameter α indicates the weighing which is given to opposite preference opinions. It is a way of requiring a level of coincidence (all or a majority of opinions agree in order to establish a preference relationship) of individual group opinions (i.e., if $\alpha = 0$, no consideration for opposite opinions is given (no agreement of opinion is required) and the ECR takes the form of a linear additive SWF; if α is large, then complete unanimity of opinion is required to establish preference relations as an opposing preference is heavily weighed). The parameter β is used to indicate the intensity (strong, weak, etc.) of the preference relations. This parameter allows the DM to differentiate the strong from the weak preference relations. It is in effect a threshold which can be set to allow only preference relations above this value to be recognized. The parameters α and β are initially set at large values and the pairwise preference relations are determined. These preference relations are transformed to a digraph as in an ISM process. If the digraph arrangement of alternatives does not have vertical structure, then the threshold β is decreased in small increments (which has the effect of establishing the weaker pairwise preference relations which before were cancelled due to the large threshold) until a vertical digraph is established or $\beta = 0$. If $\beta = 0$ and a vertical digraph is still not evident, then the coincidence of opinion parameter, α , is decreased and the algorithm repeated until a vertical or near vertical digraph results. The ECR method was also deemed applicable to EWARD because of the previous discussion of situations where occasionally certain DMs decide to not take part, or to delay taking part in the decision process. Through the multiple independent entity/MCDM refinement technique (Banker and Gupta, 1978) this procrastinating DM's preference can be bypassed by his or her specific group preferences, and surrogately included through the weights w_{yz}. Also adjustments made in w_i, and α could be used to motivate the DM into taking a prompt active role.

The weights shown in Table 12 were elicited from each group to establish the intergroup comparison of utilities. This ECR algorithm was programmed on the digital computer for efficiency because of all the

Table 12

Intergroup Weighing Of Utilities

	w_{i1}	w_{i2}	w_{i3}	
Group 1	0.38	0.41	0.21	$\sum_j w_{ij}=1$(normalized)
Group 2	0.25	0.50	0.25	
Group 3	0.25	0.25	0.50	
average w_j ($w_j=\sum_i w_{ij}/3$)	0.29	0.39	0.32	

- w_{ij} is the weight assigned to the utility of Group j by Group i
- w_j is the resulting average weight of Group j

required alternative comparisons. Information from Tables 11b, and 12 were used along with various values of α and β to establish the preferences shown on Table 13 with resulting digraphs on Figure 16.

Results

A digital computer coding of the ECR method gave a binary preference output for each pairwise set of alternatives which was converted through an ISM format (Table 13) to the digraphs of Figure 16. For the various values of α and β, Figure 16 shows that there are three alternatives which rank consistently higher than the other alternatives. These alternatives (1, 4, and 8) exhibit medium or compromise values of attributes of cost and EW performance along with high aircraft performance. For α (the parameter which weighs opposite preference opinions heavily) in the range of $0 \leq \alpha \leq .095$ and $\beta = 0$ (with $\alpha = 0$, the ECR social choice function assumes the form of a linear additive SWF) alternative 8 dominates (Figure 16a). As α is increased to larger values, the alternatives aligned themselves in two groups of a preferred set (1, 4, and 8) and a dominated set (7) as illustrated in Figure 16b and 16c. As a threshold value was brought in (β increases from 0), the ranking of Figure 16a started to decompose until the strengths of all preference relations are overcome and complete unaniminity would be required to establish a preference relation (Figure 16d). When these results were presented to the advisors and DMs in all groups, there was concensus that alternative 8 was the most desirable and its specifications would be used as the standard in the RFP.

Sensitivity Analysis

A sensitivity analysis was conducted to examine the robustness of the final ranking of alternative systems, and to point out critical areas in this approach to EWARD.

As a check of the DM's consistency in the MOOT/MAUT process, the utility based ranking of alternatives was made just prior to aggregating the group utilities into a joint SCF. The group utility was calculated for all alternatives not in the NDSS. The combined group utility for

Table 13

Preference Subordination Matrices

Alternatives

	1	4	7	8
1	0	1	0	1
4	0	0	0	1
7	1	1	0	1
8	0	0	0	0

$0. \leq \alpha \leq 0.095$; $\beta = 0.0$

(a)

Alternatives

	1	4	7	8
1	0	0	0	1
4	0	0	0	0
7	1	1	0	1
8	0	0	0	0

$\alpha = 1.0$; $\beta = 0.0$

(b)

Alternatives

	1	4	7	8
1	0	0	0	0
4	0	0	0	0
7	1	1	0	1
8	0	0	0	0

$\alpha \geq 15$; $\beta = 0.0$
$\alpha = 1.0$; $\beta = .01$
$\alpha = 0.2$; $\beta = .1$

(c)

Alternatives

	1	4	7	8
1	0	0	0	0
4	0	0	0	0
7	0	0	0	0
8	0	0	0	0

$\alpha = 1.0$; $\beta \geq 0.05$

(d)

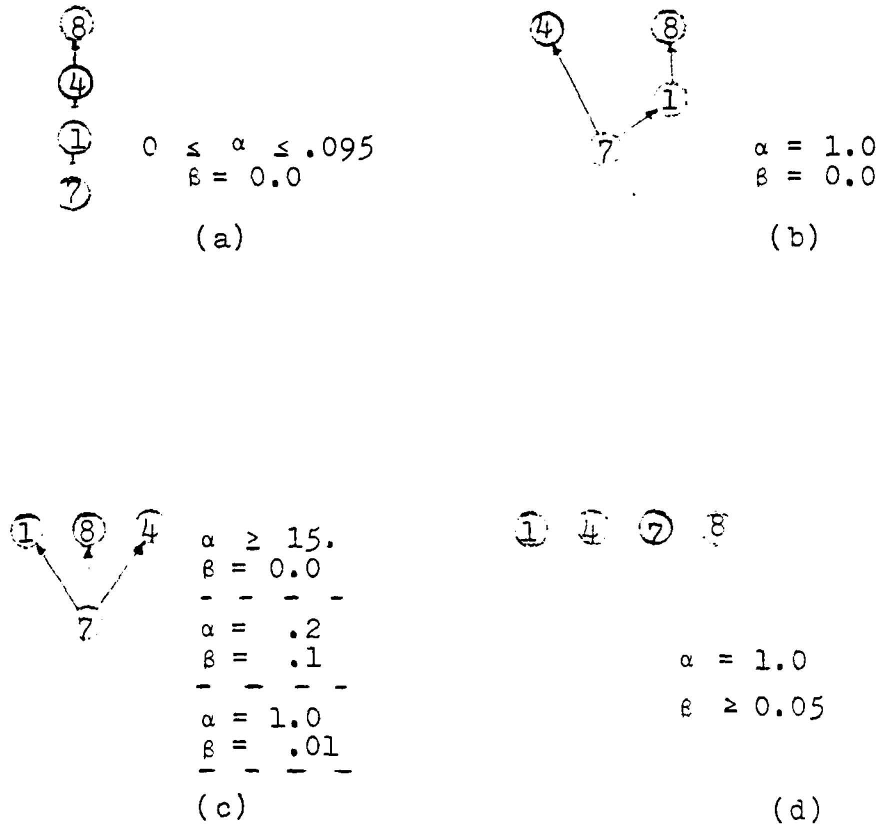


Figure 16. Social Choice Ranking Of Alternatives Using ECR

these alternatives were calculated and compared to those in the NDSS. The resulting rankings showed that alternatives 2, 3, 5 and 6 are indeed dominated in all groups. This indicates that the DMs were consistent in the value scoring of alternatives, elimination by aspects, and utility elicitation tasks.

The significance of the accuracy of the input data on the ranking of the alternatives was tested by varying the supplied values of Table 5. Varying the values of all the attributes by five percent for each alternative first in a beneficial and then detrimental direction (which had the same effect as varying the distributions over the attribute values) did not affect the groups' preference ordering of the top three alternatives. Varying all attribute levels by five percent in a beneficial direction for the second best alternative, caused this alternative (#4) to become optimal. Changing the life cycle cost attribute by six percent caused a decision switch from alternative 8 to alternative 4. These facts should alert the analyst to the realization that while alternative system parameter accuracy is important, there may be marginal returns to committing excessive resources to insure that the data is extremely accurate. This is particularly significant when considering that the data for the policy satisfaction attribute was obtained primarily through subjective estimates from the DMs and their advisors.

The sensitivity of the preferred solution to the individual group utility functions was next examined. The individual utility functions for the attributes, and the individual group utility functions for each group were varied by several percent up to five percent. The results showed that variations up to five percent for the constants in the individual attribute utility functions did not cause a change in the rankings of the alternative systems in the NDSS. Conversely, changes in the scaling constants in the amalgamated utility functions for the individual groups of only three percent caused variation in the ranking order of the top three alternatives for all groups. These results suggest that the analyst needs to devote adequate efforts to ensure accuracy in scaling constants for the combined utility functions. This was an indication of the need for importance of careful amalgamation of utilities in each

group through the use of a technique like the multiple independent entities algorithm (Banker and Gupta, 1978).

A check on the effects of the variation of the individual group's stated minimum acceptable attainment level was accomplished next since these attainment levels were the basis for an elimination by aspect exercise. It is noted that because all attainment levels of each individual group had to be justified to the other groups, only reasonable and fairly conservative attainment levels were supplied by the groups. While variation by twenty percent in several attainment levels (policy satisfaction, threats degraded, etc.) caused variation in the membership of alternatives in the NDSS, these affected alternatives (5 and 7) were marginal and not prominently ranked in the final analysis.

Lastly, the robustness of the selection of the optimum alternative system was examined with respect to the weighing of each other's opinion by the individual groups. In the ECR group utility aggregation algorithm, the intergroup opinion weights were varied up to ten percent with only minor variation in the final alternative rankings (alternative system 4 and alternative system 1 changed places in the rankings). Alternative 8 was still the optimum choice even with the aforementioned variation in the intergroup comparison of utilities.

Validation of the Decision Procedure

The DMs and advisors who took part in this exercise expressed satisfaction in the MOOT/MAUT approach to EWARD. Therefore, this technique was used in a validation exercise to see how it would have done on an actual system which is operational now. The data from a recent EW retrofit system was processed in a pre-analysis phase (Section 3) to fit the modelling approach of the combined MOOT/MAUT method presented in this chapter. Four alternative systems were reduced in the three groups (G-1, G-2, G-3) to a set of non-dominated alternatives from which the groups selected a "best" alternative. The alternative which was selected was a modification of the system which was actually retrofit on the aircraft. All groups agreed that the

system selected by the MOOT/MAUT approach was superior, and was selected in a more efficient manner compared to the system which was eventually retrofit. The DMs expressed their opinions for the discrepancy and these can be summed up in two points:

1. There was no comprehensive set of criteria (such as on Table 1) which could be used to judge system goodness early in the acquisition life cycle in the actual retrofit.

2. There was a lack of communication between pertinent stakeholders in the actual retrofit which was significantly ameliorated by the MOOT/MAUT technique. Therefore, it is concluded that this exercise validates the appropriateness and efficiency of the use of MCDT (specifically the MOOT/MAUT approach) in the early phases of the DOD Equipment Acquisition Life Cycle.

Method Acceptability

In this effort, twenty-one DMs and advisors from the various stakeholder groups (Appendix C) took part. Interviews with these participants consisted of fact finding and preference elicitation sessions. The average time spent with each participant was 2.4 hours, and the maximum time for any individual was 18 hours. The majority of participants (nineteen) expressed satisfaction and acceptance of this approach to EWARD, and indicated they would be in favor of using this technique in future efforts. The majority of participants gave the opinion that the two main benefits of this approach to EWARD are:

1. The final product is an acceptable, cost effective system.

2. There is a definite savings in time accrued through this approach (the participants estimated that to one years time (50%) in the Conceptual Phase could be saved utilizing this technique) compared to the present design procedure.

Implementation

As mentioned in Section 2 and 3, the application of the MOOT/MAUT approach to EWARD should be in the Conceptual Phase of the Defense

Systems Acquisition Cycle before the PMP is prepared. The application at this point in the acquisition cycle would allow for a maximum of benefits (determine the system configuration and get all pertinent groups communicating) without the need for legislation, or any changes of the current regulations. The current regulations do not generally specify application of particular techniques, but they allow for the application of desirable techniques. The analysts required could be drawn from the staffs in all three groups (user Commands, Hq. USAF and/or DOD, and Systems/Logistics Commands), and formed into a dedicated team for the EWARD (verses working at the retrofit problem in fragmented groups as they do currently) without the requirement for additional personnel. This team would ideally be responsible to the DSARC/SECDEF to allow the team to operate with the cooperation of all groups but immune from the influence of any one group.

5. Summary

In this effort, a decision situation involving Electronic Warfare Aircraft Retrofit Design (EWARD) was modelled and resolved using the combined MOCT/MAUT approach. The chapter began by an overview of the DOD Equipment Acquisition Cycle as set down in various regulations and directives. Esoteric considerations in the electronic warfare community were described because of difficulties they cause in the acquisition procedure. The primary stakeholder groups were identified along with their interaction in the equipment retrofit situation. The decision making procedure was applied to the Conceptual Phase of the DOD Equipment Acquisition Cycle where it was intended to produce two results:

1. A salient set of design criteria (candidate system descriptors) which can be used in judging the goodness of a proposed EW alternative design in the early phases of the EW equipment acquisition.

2. A decision making procedure for candidate system selection as a basis for system selection and the RFP.

The EWARD situation was modelled around attributes adequate for judging the EW retrofit system and a set of realistically based alternatives for a retrofit design. These alternatives were supplied with

attribute levels and the associated risk for realization of the alternatives. Using the MCDT approach, the combined MOOT/MAUT process was used to solve the EWARD situation. A pre-analysis phase was used to structure the decision situation. The optimization step consisted of a multi-dimensional elimination by stochastic dominance exercise since the impacts of implementing the various alternative systems are provided by the competing contractors. The resulting NDSS was further reduced in the number of alternative systems when the individual groups supplied minimum acceptable attainment figures for the attributes as an input to an elimination by aspects exercise. The MAUT technique of multiattribute decision analysis was used to model and rank the alternatives in the NDSS by developing a cardinal utility SCF for each group. These group utility functions were amalgamated into an ultimate SCF which produced a final ranking of the NDSS and identification of alternative 8 (Table 5) as the optimum system configuration. Group concensus corroborated this alternative selection. A certain amount of iteration was required in all steps of the MOOT/MAUT process in order to converge to an acceptable policy choice.

A subsequent sensitivity analysis of the final ranking of the alternatives (with alternative 8 as the identified optimal configuration) produced the following observations: the DMs were consistent with respect to value scoring and utility function scoring of the attribute as a basis for forming the NDSS and then ranking the resulting alternatives; the final ranking of alternatives was somewhat sensitive to the accuracy of the alternatives systems impact data, and the stated minimum acceptable attainment levels of the individual groups; the final ranking was very sensitive to the accuracy of the constants in the individual group aggregated utility functions. A validation exercise was then carried out using data from a recent EW retrofit system. The MOOT/MAUT approach described earlier was used in the pre-analysis, modelling and identification of the optimum system configuration. The groups selected a system by concensus which all DMs and advisors stated was the superior system (the system was a modification of the actual system now in the aircraft) of the alternatives available.

We have illustrated a need for and the use of a MCDT approach in the early phases of the DOD EW equipment acquisition effort. The MOOT/MAUT approach can efficiently produce a solution to the EWARD situation which contains risk/uncertainty elements. The validation effort demonstrated that such an approach forces DMs and their advisors to consider the salient attributes early and to communicate (or at least add opinions and inputs to the situation) with other stakeholder groups. The MOOT/MAUT technique efficiently defines the most desirable alternative available. The MOOT/MAUT technique includes the DMs in many steps in the technique which should give the DMs confidence in the results obtained. The multiplicative form of scoring function for evaluation of alternative configurations is an appropriate form in EWARD when the relationships of the attributes and the desire for non-compensatory policies are incorporated into this decision situation.

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CHAPTER 6

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

1. Summary

In this effort, comparison of two multiple criteria decision techniques (MOOT and MAUT) motivated the generation of a more efficient decision support methodology. This combined MOOT/MAUT method aggregates the complementary aspects of both approaches. A simple example of the approach was followed by a detailed application to a Department of Defense equipment acquisition situation involving the retrofitting of a special purpose aircraft. Decision makers and advisors from identified primary stakeholder groups took part in the EWARD application and subsequent validation exercise. A framework for electronic warfare aircraft retrofit was developed using the combined multi-criteria MOOT/MAUT process. A set of criteria or attributes was developed in the EWARD application which can be used to evaluate alternative system configurations in a comprehensive manner throughout the equipment acquisition cycle.

The major contributions of this effort are the delineation of the systems engineering process algorithms for MOOT and MAUT(pp23-32,49-56), the development of a combination MOOT/MAUT methodology based on the complementary characteristics of both approaches (pp 69 - 77), the development of an efficient framework for EWARD through the extension of a MCDT to that application (pp 98 - 143), and the generation of a set of criteria for evaluation of alternative retrofit EW systems in the Defense Systems Acquisition Cycle (pp 98-143).

2. Conclusions

Based on this research, the following conclusions can be drawn:

- While there are operational and philosophical differences between MOOT and MAUT, both processes are mental constructs to approaching decision situations, and when they are compared at the same level, there should be no essential differences in structure between them. At the application level, both MOOT and MAUT approaches seek

identification of a strategically equivalent optimal policy, assuming the DM is consistent and the NDSS is complete.

- The complementary phases of MOOT and MAUT are compatible for combination into a single methodology.

Specific conclusions which can be drawn from the EWARD application are as follows:

- A MCDM approach has merit in an EWARD application, particularly in the early stages of this application. The combined comprehensive resource MOOT/MAUT approach will increase the efficiency of EWARD in an overall time and resource savings.
- The multiplicative form of scoring function for evaluation of alternative configurations is an appropriate form in EWARD to insure sensitivity of the scoring form to small differences in the alternatives (pp 121 to 129, A-9 to A-12).
- The EWARD framework presented is an acceptable and desirable approach to this specific defense systems equipment acquisition situation according to the decision makers in the identified primary stakeholder groups.
- Careful assessment of preferences,and corroboration of the scaling constants in the aggregated utility functions of DMs and advisors is critical to identify the optimal system configuration correctly.

3. Recommendations

From the research and application efforts, the following recommendations can be offered:

- The combined MOOT/MAUT methodology is feasible for implementation in EWARD in the Conceptual Phase of the Defense Systems Equipment Acquisition Cycle. This process should be implemented utilizing the available analytic and technical staffs of the identified stakeholder groups. For convenience, it is suggested that these staffs should be organized in a single team mode under the direction of SECDEF/DSARC.
- The set of developed technical criteria should be utilized in EWARD as a comprehensive means to evaluate retrofit system configurations.

As a result of our efforts, we can identify the following areas of further research. There is a need to further develop techniques for selection of the optimal policy from a non-dominated solution set in ways that facilitate decision maker-analyst interaction. On the whole, this important area has received little effort from the MOOT advocates. Another area for further investigation concerns the need for more adequate modelling of the behavioral aspects of decision situations so that by incorporating a DM's cognitive style, there would result increased DM acceptability of the analytic approach and the results. Additionally, research aimed at examining mathematically, the preference space at pre-optimization (for MAUT) and post-optimization (for MOOT) is a direction for further efforts which would aid in the steps of modelling,ranking alternatives, and decision making for multiple criteria decision situations. Efforts are also needed to further characterize the stated objective of national policy satisfaction as it relates to EWARD.

APPENDIX A

ON WEAKENED SUFFICIENCY REQUIREMENTS FOR THE MULTIPLICATIVE FORM OF VALUE FUNCTION

1. Introduction

Mutual preferential independence of attributes is generally used as a sufficiency condition for the multiplicative form of value function. Often the effort to verify existence of this condition is considerable. In this appendix, new and weakened sufficiency conditions are given for the multiplicative form of value function. Use of these new conditions will generally require much less effort than use of the mutual preferential independence conditions. Also the new sufficiency conditions will often present very desirable measurement sensitivity conditions.

Section 2 discusses the motivation for use of multiplicative form of value function. Section 3 presents and proves the existence of weaker conditions which allow use of the multiplicative function while alleviating much of the verification process. Finally, section 4 presents a numerical application which concerns aircraft retrofit requirements.

2. The Additive Value Function

The popularity of the additive form of measureable value functions is warranted because of the facility of assessment of the constituent value functions and ease of evaluation of the scaling constants (Keeney and Raiffa, 1976; Dyer and Sarin, 1979). Unfortunately, two drawbacks should, in practice, often discourage the widescale use of the additive form of value function. The additive form of value function is

$$v(\underline{x}) = \sum_{i=1}^{n} k_i \cdot v_i(x_i) \qquad \text{(A-1)}$$

where $v(x)$ is a scalar signifying the score for a specific action with attribute levels x_i, $v_i(x_i)$ is a component value statement for attribute level x_i, $\underline{x}$ is an n vector of attribute levels, and k_i is a scaling constant. It is proper to use this additive form only when the

attributes, x_i, are mutually preferentially independent (MPI) (Keeney and Raiffa, 1976), and this can be difficult for the gerent to verify. It is a sizable task to show MPI of attributes where the dimension of the value attributes, n, is large.

Another shortcoming of the additive form is that it tends to be insensitive to individual attribute levels. Additive forms are compensatory in the sense that an increase in one attribute can compensate for a decrease in any other attribute. This means that large increases or decreases in any one attribute may be offset by changes in other attribute levels and consequently may have little effect on the scoring value of the total model. Huber and Johnson (1977) have pointed out that this compensatory characteristic may be undesirable in many applications.

3. The Multiplicative Value Function

A form of value function which alleviates these difficulties is the multiplicative form. While the requirement for MPI of attributes is a sufficient condition for a multiplicative form of value function, it is not a necessary condition as it was for the additive form. Dyer and Sarin (1979), building on the work of Fishburn (1976) and Keeney and Raiffa (1976), have determined a sufficient condition for the multiplicative form of the general multilinear form. If attributes $X_1, X_2, , , X_n$ are mutually weak difference independent (MWDI), then the form of the value function is:

$$v(\underline{x}) = \sum_{i=1}^{n} k_i v_i(x_i) + K \sum_{\substack{i=1\\j>i}}^{n} k_i k_j v_i(x_i) v_j(x_j)$$

$$+K^2 \sum_{\substack{i=1\\j>i\\l>j}}^{n} k_i k_j k_l v_i(x_i) v_j(x_j) v_l(x_l) \ldots . + K^{n-1} \prod_{i=1}^{n} k_i v_i(x_i) \tag{A-2}$$

or

$$1 + Kv(\underline{x}) = \prod_{i=1}^{n} [1+Kk_i v_i(x_i)]. \tag{A-3}$$

To continue the development, it is desirable to define weak difference independence (WDI).*

Definition: Attribute X_i is WDI of $X_{\bar{i}}$ if given any a_i, b_i, c_i, $d_i \varepsilon X_i$, such that $v(a_i, a_{\bar{i}}) - v(b_i, a_{\bar{i}}) \geq v(c_i, a_{\bar{i}}) - v(d_i, a_{\bar{i}})$ for some $a_{\bar{i}} \varepsilon X_{\bar{i}}$, then it is required that $v(a_i b_{\bar{i}}) - v(b_i b_{\bar{i}}) \geq v(c_i b_{\bar{i}}) - v(d_i b_{\bar{i}})$ for any $b_{\bar{i}} \varepsilon X_{\bar{i}}$.

Thus WDI means that the ordering of preference differences depends only on the value differences associated with attribute X_i and not on the fixed value of all other attributes of $X_{\bar{i}}$. The attribute X_i is WDI of $X_{\bar{i}}$ if the value function is of the form

$$v(X_i, X_{\bar{i}}) = g(X_{\bar{i}}) + h(X_{\bar{i}})\, v(X_i, X_{\bar{i}}') \qquad \text{(A-4)}$$

for all $X_i, X_{\bar{i}}$, and $X_{\bar{i}}'$, where $g(X_{\bar{i}})$ and $h(X_{\bar{i}}) > 0$ are functions which depend only on $X_{\bar{i}}$.

A physical interpretation of this definition of WDI will now be presented for an equipment retrofit design effort. It is assumed that only two attributes are important to the problem, X_1 which represents volume and $X_2 = X_{\bar{1}}$ which represents cost. The object is to minimize the cost and the volume of the retrofit equipment. To check if X_1 is WDI of $X_{\bar{1}}$, we first choose levels of $a_1, b_1, c_1, d_1 \varepsilon X_1$ and $a_{\bar{1}} \varepsilon X_{\bar{1}}$, so that the exchange of the combination of attributive levels $(b_1, a_{\bar{1}})$ for the pair $(a_1, a_{\bar{1}})$ is preferred to the exchange of pair $(d_1, a_{\bar{1}})$ for $(c_1, a_{\bar{1}})$. If this preference order is preserved for other levels of attribute $X_{\bar{1}}$, that is to say if the exchange of $(b_1, b_{\bar{1}})$ for the combination $(a_1, b_{\bar{1}})$ is preferred to the exchange of pair $(d_1, b_{\bar{1}})$ for $(c_1, b_{\bar{1}})$ for any $b_{\bar{1}} \varepsilon X_{\bar{1}}$, then X_1 is WDI of $X_{\bar{1}}$. In this case let $a_1 = 4m^3$, $b_1 = 5m^3$, $c_1 = 2m^3$, $d_1 = 3m^3$ and $a_{\bar{1}} = \$10^6$, and assume that $4m^3$ equipment volume fills the allotted space and that $5m^3$ of equipment means a cutback of a co-located equipment function. To satisfy the above requirement at a single attribute level, the decision maker (DM) must prefer the exchange of a system with characteristics of $(5m^3, \$10^6)$ for a configuration $(4m^4, \$10^6)$

*The symbol $Y_{\bar{i}}$ is used to indicate all components of Y not contained in Y_i.

over the exchange of a configuration ($3m^3$,$\$10^6$) for a configuration ($2m^3$,$\$10^6$). WDI requires that this preference order for exchanges must apply for other levels of $X_{\bar{1}}$ such as $b_{\bar{1}}$ = \$1.5 X 10^6 in each system configuration. Thus the DM must still prefer the exchange of system ($5m^3$,\$1.5 X 10^6) for system ($4m^3$,\$1.5 X 10^6) over the exchange of system ($3m^3$,\$1.5 X 10^6) for system ($2m^3$,\$1.5 X 10^6). If the DM expresses the same preference order responses for a number of different quadruples of levels X_1 and fixed levels of $X_{\bar{1}}$, then it can be deduced that X_1 is WDI of $X_{\bar{1}}$. A caveat should be issued in order to insure that the DM is stating preferences concerning exchanges of configurations and outcomes, and not stating preference for the configurations or outcomes themselves. For instance a rational DM who prefers an exchange of the pair (\$100, 1 ounce of gold) for (\$105, 1 ounce of gold) to an exchange of the attribute pair (\$40, 1 ounce of gold) for (\$50, 1 ounce of gold) is obviously not concentrating on the exchange itself (if it is assumed that the DM has a linearly increasing monetary value preference curve) because this DM is irrationally preferring an increase in cash of \$5 to an increase of \$10. A DM who incorrectly stated this would be erroneously establishing the first condition for WDI at the single attribute level. This exchange preference idea agrees with Kahneman and Tversky (1977) in their Prospect Theory which accounts for the reference effect of the asset position of the DM in rational choices.

The verification of the appropriateness of the multiplicative form of value function require checking all subsets of attributes of WDI. We present the following to make this task simpler and less time-consuming.

Theorem 1. Given attributes $X_1, X_2, , , X_n$, the following are equivalent:

a) Attributes $X_1, X_2, , , X_n$ are mutually weak difference independent (MWDI).

b) X_i is weak difference independent of $X_{\bar{i}}$, and (X_i, X_j), $j \neq i$, is preferentially independent (PI); $j = 1,2,3,,,n$; $n \geq 3$.

The result of this theorem allows a much reduced effort in order to verify sufficient conditions for the validity of a product form of measurable value function. The proof of this theorem requires a fundamental relationship between PI and WDI. This relationship follows

the same reasoning presented in Keeney (1974) and Keeney and Raiffa (1976) for weak sufficiency associated with mutual utility independence.

Our proof is simplified by consideration first of the three attribute cases described by Lemma 1:

Lemma 1. Given a set of attributes A, B, and C; if A is weak difference independent (WDI) of $\bar{A}$, and if (A,B) is preferentially independent (PI) of $(\bar{A},\bar{B})$, then (A,B) is WDI of $(\bar{A},\bar{B})$.

The proof of this lemma proceeds as follows. We let $\bar{A} = B\theta C$ where θ signifies a Cartesian product space. The case where (A,B) is WDI of $(\bar{A},\bar{B})$ for all pairs of attributes is a sufficient condition for the proof of Theorem 1 in the three attribute cases, and this can be extended to the n attribute case, for mutual weak difference independence (MWDI) (Dyer and Sarin, 1979). The condition where A is WDI of $\bar{A}$ can be represented by

$$v(a,b,c) = g(a^0,b,c) + h(a^0,b,c)v(a,b^0,c^0), \quad h > 0 \qquad \text{(A-5)}$$

where a, b, and c represents levels of attributes A, B, and C respectively. We assume that the function $g(\cdot)$ is also a measurable value function with the same mapping as $v(\cdot)$. Therefore for simplicity, we replace $g(a,b,c)$ by $v(a,b,c)$. The function $h(\cdot)$ is defined as a positive value function similar to $v(\cdot)$. Since (A,B) is PI of C, we know that

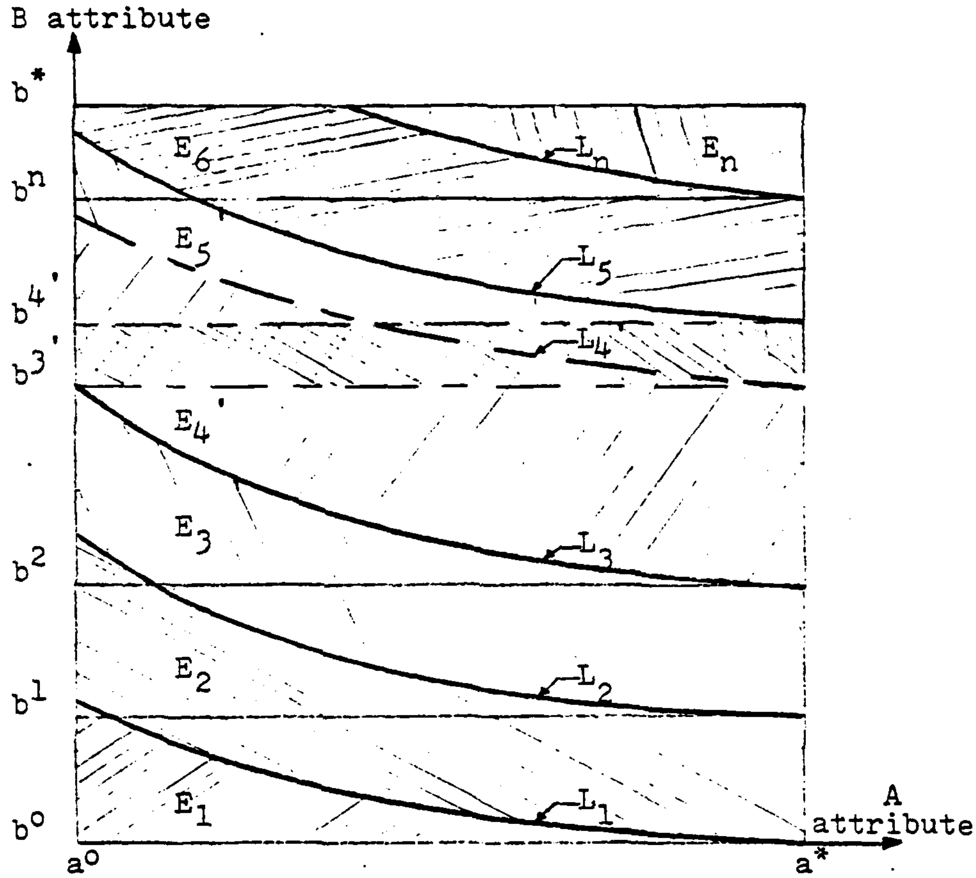
$$v(a^2,b^2,c^2) \geq v(a^1,b^1,c^2)$$

implies

$$v(a^2,b^2,c^1) \geq v(a^1,b^1,c^1), \quad \forall c \varepsilon C. \qquad \text{(A-6)}$$

Proof of this lemma above requires one to ascertain that (A,B) is WDI of C.

Constructs motivating our proof can be illustrated graphically as in Figure A-1. Here a and b are scalar attributes. It is first shown that the condition of (A,B) being WDI of C holds for all c and (a,b) pairs in E_1. Then because a horizontal line $b = b^1$ intercepts indifference line L_1 and regions E_1 and E_2, this allows the WDI condition to hold for all pairs (a,b^1). Now other (a,b) pairs in E_2 are indifferent to the pair (a,b^1). This extends the WDI concept to region E_2. This same procedure is repeated over and over again until all of the attribute space is covered. Then it is shown that all pairs $(a,b) \varepsilon (A,B)$ are WDI



L_i = isopreference curve i
E_i = trade-off area i

Figure A-1. Conceptual Illustration of Requirements for Difference Independence

$c \varepsilon C$ and use of the multiplicative form of value function given by Eq. (A-2) or (A-3) is justified.

Formal proof of Lemma 1 is now a relatively simple matter. A pair (a,b) in E_1 is defined by

$$E_1 = \{ (a,b,c^0): v(a,b,c^0) \leq v(a^*,b^0,c^0)\} \quad , \tag{A-7}$$

we assume there exists an a^1 such that

$$v(a,b,c^0) = v(a^1,b^0,c^0) \quad \forall (a,b) \varepsilon E_1 \tag{A-8}$$

Now from Eqs. (A-6) and (A-7), it follows that

$$v(a,b,c) = v(a^1,b^0,c), \quad \forall c \varepsilon C, \ (a,b) \varepsilon E_1 \tag{A-9}$$

Substituting Eq. (A-9) into Eq. (A-5) results in

$$v(a,b,c) = v(a^0,b^0,c) + h(a^0,b^0,c)v(a^1,b^0,c^0) \ \forall c \varepsilon C, \ (a,b) \varepsilon E_1 \tag{A-10}$$

We now combine Eqs. (A-8) and (A-10) to eliminate $v(a^1,b^0,c^0)$ and this yields the desired result

$$v(a,b,c) = v(a^0,b^0,c) + h(a^0,b^0,c)v(a,b,c^0), \quad \forall c \varepsilon C, \ (a,b) \varepsilon E_1 \tag{A-11}$$

As Eq. (A-11) shows, the WDI condition is shown for (A,B) worth independent of C in the region E_1. To extend this for all possible (a,b) pairs in space $A\theta B$, we shall next move into space E_2. There, we choose b^1 such that

$$v(a^0,b^0,c^0) < v(a^0,b^1,c^0) < v(a^*,b^0,c^0) \tag{A-12}$$

Since $(a^0,b^1) \varepsilon E_1$, we may replace a by a^0 and b by b^1 in Eq. (A-11) to obtain

$$v(a^0,b^1,c) = v(a^0,b^0,c) + h(a^0,b^0,c)v(a^0,b^1,c^0), \quad \forall c \varepsilon C \tag{A-13}$$

We rewrite Eq. (A-5) using the levels b^1 and c^0 levels as

$$v(a,b^1,c^0) = v(a^0,b^1,c^0) + h(a^0,b^1,c^0)v(a,b^0,c^0) \quad \forall a \varepsilon A. \tag{A-14}$$

Now we set $b = b^1$ in Eq. (A-11) to obtain

$$v(a,b^1,c) = v(a^0,b^0,c) + h(a^0,b^0,c)\ v(a,b^1,c^0), \quad \forall (a,b^1) \varepsilon E_1 \quad \text{(A-15)}$$

This result can now be combined with Eq. (A-14) to yield

$$v(a,b^1,c)=v(a^0,b^0,c)+h(a^0,b^0,c)\ [v(a^0,b^1,c^0)+h(a^0,b^1,c^0)v(a,b^0,c^0)]$$

$$=v(a^0,b^1,c)+h(a^0,b^0,c)h(a^0,b^1,c^0)v(a,b^0,c^0),\ \forall c\varepsilon C,\ (a,b^1)\varepsilon E_1 \quad \text{(A-16)}$$

We now use the inequality of Eq. (A-12) to define and restrict b^1. There exists an a with $v(a,b^0,c^0 > v(a^0,b^0,c^0)$ which satisfies Eq. (A-16). We compare Eq. (A-16) to Eq. (A-5) with $b = b^1$ and this shows that

$$h(a^0,b^1,c) = h(a^0,b^0,c)h(a^0,b^1,c^0), \quad \forall c\varepsilon C. \quad \text{(A-17)}$$

Substituting Eqs. (A-13) and (A-17) into Eq. (A-5) with $b = b^1$ results in

$$v(a,b^1,c)=v(a^0,b^1,c)+h(a^0,b^1,c)v(a,b^0,c^0) = v(a^0,b^0,c)$$

$$=v(a^0,b^0,c)+h(a^0,b^0,c)\ [v(a^0,b^1,c^0)+h(a^0,b^1,c^0)v(a,b^0,c^0)] \quad \text{(A-18)}$$

now we combine Eqs. (A-18) and (A-14) to obtain

$$v(a,b^1,c) = v(a^0,b^0,c) + h(a^0,b^0,c)v(a,b^1,c^0), \quad \text{(A-19)}$$

Region E_2 is defined by

$$E_2 = \{(a,b,c^0):v(a^*,b^0,c^0)<v(a,b,c^0)\leq v(a^*,b^1,c^0)\} \quad \text{(A-20)}$$

For any $(a,b)\varepsilon E_2$, there exists an a^2 such that

$$v(a,b,c^0) = v(a^2,b^1,c^0), \quad \forall c\ \varepsilon\ C,\ (a,b)\varepsilon E_2 \quad \text{(A-21)}$$

Consequently from Eq. (A-6), it follows that

$$v(a,b,c) = v(a^2,b^1,c), \ (a,b)\ \varepsilon\ E_2. \quad \text{(A-22)}$$

Now we evaluate the right hand side of Eq. (A-22) using Eq. (A-19) to obtain

$$v(a,b,c) = v(a^0,b^0,c) + h(a^0,b^0,c)\ v(a^2,b^1,c^0) \quad \forall c\varepsilon C \quad \text{(A-23)}$$

when combined with Eq. (A-21) the foregoing yields

$$v(a,b,c) = v(a^0,b^0,c) + h(a^0,b^0,c)\ v(a,b,c^0),\ (a,b)\varepsilon E_2 \tag{A-24}$$

This shows the desired result that (A,B) is WDI for regions E_1 and E_2. This same process can be and is repeated until the entire attribute space is covered. Additional isopreference lines may need to be inserted to allow overlap of the indifference regions for given attribute levels of the attribute space. This is shown by interaction of L_4', $b^{3'}$. and $b^{4'}$ in Figure A-1. The continuity assumption on the measurable value function v, and the non-satiation assumption that more is better than less of a desired attribute; or that less is better than more of an undesired attribute, and the assumption that h(a,b,c) is positive allows one to show that (A,B) is WDI of C.

The preceding weakened conditions for establishment of MWDI (and resulting verification of the multiplicative form of value function) ameliorate drawbacks associated with verifying the additive form of value function. Figure A-2 illustrates an assessment procedure to establish MWDI of the attributes. This lessens the time commitment required of the DM and analyst. Also it provides a less compensatory scoring form, because of the product terms that allows for more sensitivity to the attributes. This is essential in certain applications.

4. Numerical Example

In the application of the approach suggested here to an electronic warfare equipment selection situation, a proposed measurable value function, which can serve as a criterion for a multiobjective optimization approach, can take the form:

$$v(\underline{x}) = f(X_1,X_2,,,X_n) \tag{A-25}$$

where the system effectiveness attribute vector takes the product form:

$$f(X_1) = v_1(X_1) = k_1X_{1a} + k_2X_{1b} + k_{12}X_{1a}X_{1b} \tag{A-26}$$

where X_{1a} = no. of threats covered by alternatives and X_{1b} = degree of

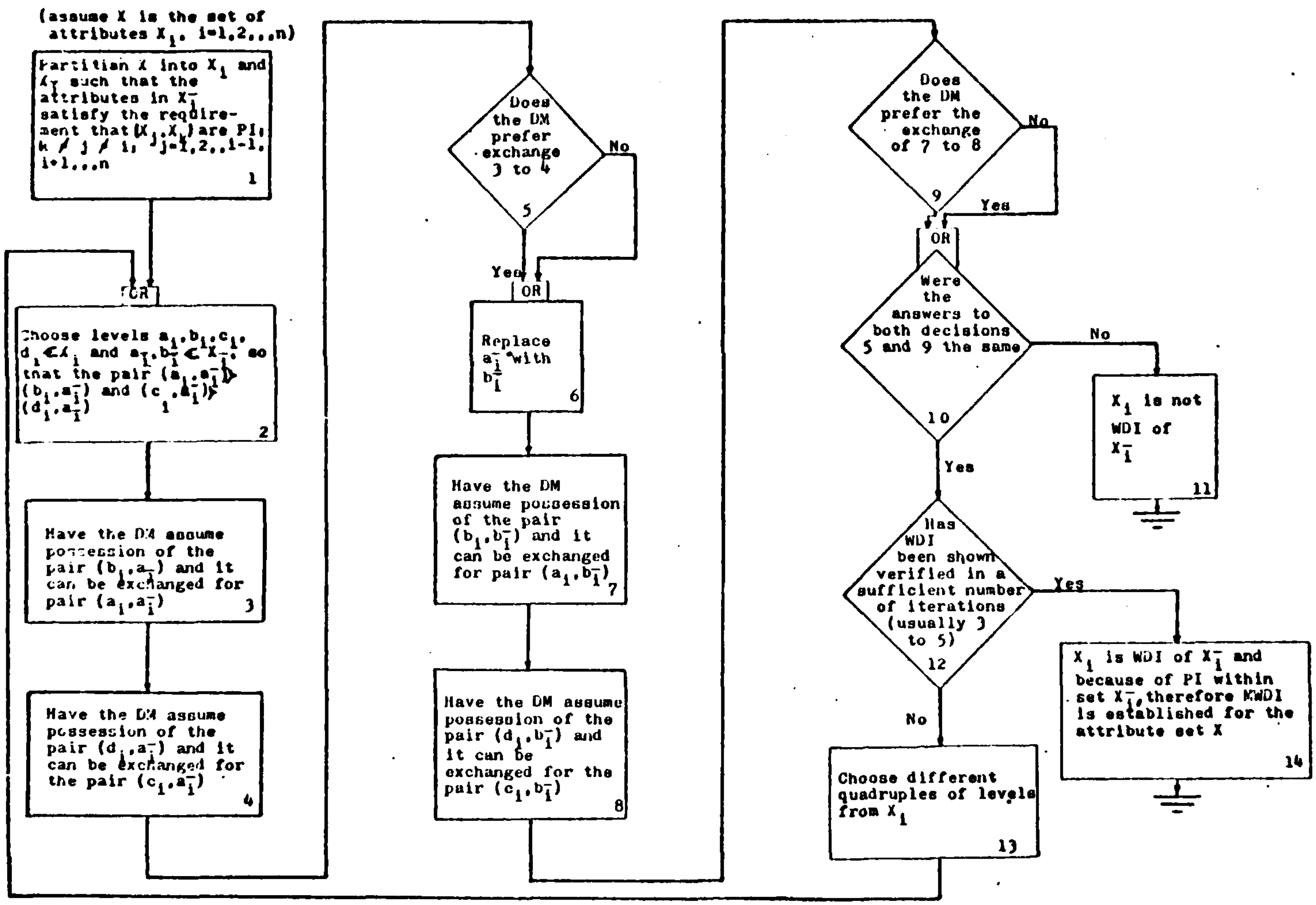


Figure A-2. MWDI Assessment Procedure

effectiveness produced by alternatives. Assuming that MWDI has been established for the two attributes, the product form above reflects the required sensitivity to alternatives with either an excess or lack of either attribute. Past use of additive value criteria in the electronic warfare area have not been sufficiently able to penalize inferior alternatives. This has led to unjustified confidence in the capability of operating systems as has been pointed out by Peterson, Hays and O'Conner (1975).

To illustrate this we consider two alternatives A and B with the following pertinent characteristics:

Alternative A: *8 out of 10 critical threats are affected by the system (score $\frac{8}{10}$ = .8)

*3 out of the 10 of the critical threats are covered sufficiently to assume them countered (score $\frac{3}{10}$ = .3)

Alternative B: *6 out of 10 critical threats are affected by the system (score $\frac{6}{10}$ = .6)

*5 out of 10 critical threats are sufficiently countered (score $\frac{5}{10}$ = .5)

Let an additive value function, assuming MPI is also established, of the form

$$v_{11}(X_1) = k_{11}X_{1a} + k_{22}X_{1b} \tag{A-27}$$

be used as a comparison with the multiplicative form of (A-26). Assume the scaling constants have been assessed to be $k_{11} = k_{22} = .5$ and $k_1 = k_2 = k_{12} = .333$ so that v_1 and v_{11} are normalized to equal 1.0 when both X_{1a} and X_{1b} are at their maximum value of 1.0 and $v_1 = v_{11} = 0.0$ when both X_{1a} and X_{1b} are at their minimum value of 0.0. Now inserting the values above (for alternative A; $X_{1a} = .8$, $X_{1b} = .3$ and for alternative B; $X_{1a} = .6$, $X_{1b} = .5$), the following results are obtained:

Alternative A: $v_{11} = .55$ and $v_1 = .45$

Alternative B: $v_{11} = .55$ and $v_1 = .47$

The results show that the additive form of v_{11} evaluate both alternatives identically whereas the multiplicative form of v_1 differentiates in favor of alternative B. As a matter of interest, all of the DM's polled, preferred alternative B which indicates that there is a strong basis for pursuing the multiplicative criteria form in certain application efforts where sensitivity is paramount.

Summary

Alternative sufficient conditions which can be used to verify MWDI of attributes for the valid use of the multiplicative form of value function have been obtained. These conditions allow for reduced efforts in this verification process which is of value to decision makers and analysts. A summary of a recommended assessment procedure has been presented. The increased sensitivity of the product form of value function make this form particularly useful in certain applications as is evidenced by the numerical example.

APPENDIX B

EQUIVALENCE OF MULTIPLE CRITERIA DECISION MAKING APPROACHES

1. Introduction

The current interest in Multiple Criteria Decision Theory (MCDT) has been prompted by the desire of analysts and decision makers (DM) to produce solutions which better reflect the DM's preferences in decision situations. The two MCDT approaches of multiple objective optimization theory (MOOT) and multiple attribute utility theory (MAUT) have enjoyed increased popularity because of the usefulness of these tools in the resolution of complex decision situations (Cochrane and Zeleny, 1973; Starr, 1977; and MacCrimmon, 1973).

The MOOT approach was developed to be a computationally efficient optimization tool. A vector of objective functions is optimized with respect to each component to form a set of non-dominated solutions. A value function is formed, based on the DM's preferences, by which the optimum is formed (Cohon and Marks, 1975). This last action of optimum selection by the DM has not received a proportional amount of attention as has the optimization techniques.

The MAUT approach was developed to be a decision making aid. A scalar function is used to combine the DM's utility for the attributes. This scalar function, which is often called a social choice function (SCF) is formed based on a set of preferences of the DM. The alternative policies are ranked with respect to maximizing the DM's utility for the outcomes of these policies through the use of this SCF.

A question of importance to the structural research in MOOT and MAUT concerns the analytic equivalence of an optimal policy produced by both approaches to a common decision situation. That is, under certain weak assumptions, the optimal (most preferred) alternative is non-dominated. Theorem 1, to be presented below, addresses this issue. A second question arises from contemplating the use of a specific MOOT or MAUT technique in an application. This item concerns the fact that the second best alternative policy may be dominated, and hence in jeopardy

of not being considered as the new optimal if for some reason the original optimal could not be implemented. Lemmas 3 and 4, as well as an example to be presented, address this latter situation.

A specific decision situation can be characterized by a set of alternate actions $(x = (x_1,,,x_n))$ in an admissible decision space, $(x \in X)$ where X is partially ordered by $\leq$ and X' is the set of all non-dominated points in X, and a vector of cardinal utility functions $u(x)$ representing the DM's felicity for the alternative actions (or the outcomes which are a result of the actions). This information is now incorporated into the format of both approaches.

MAUT

The multiattribute utility case considers combining n individual utility function optimizations

$$\max_{x \in X} \quad u_i(x), \; u_i \in u; \; u: X \to R^n \,, \qquad (B-1)$$

(where n is a positive integer) into a social choice function U, $\exists U \ni U: R^n \to R$ to come up with a scalar ranking form and solution

$$\max_{x \in X} \quad U(u_1(x),,,u_n(x)) = \max_{x \in X} \; U(u(x)) = D_{MAUT} \qquad (B-2)$$

MOOT

The multiobjective optimization case considers first forming a vector of value functions

$$v(x) = (v_1(x),,,v_m(x)); \; v \in V \qquad (B-3)$$

and optimizing with respect to each component to produce a non-dominated solution set (NDSS) $V' \subset V \ni X' \subset X$ and $x \in X$. Then we shall say that $\hat{x}$ dominates x if $\to \nexists \; \hat{x} \in X'$

$$v_i(\hat{x}) > v_i(x) \text{ for some } v_i; \; i = 1,,,n \qquad (B-4)$$

$$v_j(\hat{x}) \geq v_j(x), \forall j \neq i \qquad (B-5)$$

Then a post-optimization process is used to incorporate the DM's utility in a social choice function for selection of the "best" of NDSS

$$\max_{x \in X} \quad U(v(x)) = D_{MOOT} \qquad (B-6)$$

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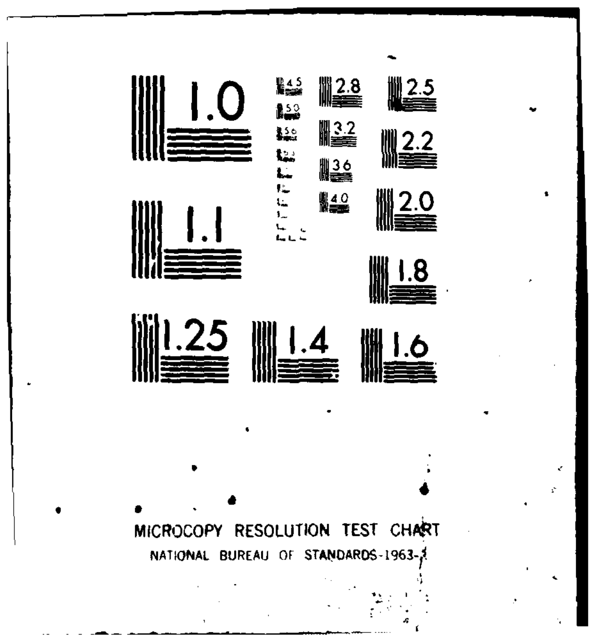
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2. Development

The question of the equivalence of D_{MAUT} and D_{MOOT} will be answered in two parts. First it is hypothesized that $D_{MAUT} = D^*$ is a global optimum of the scalar choice function U based on the DM's utility for the salient attributes. Second, it is shown below, that under weak conditions, the global optimal policy D_{MOOT} is also the optimum of the NDSS (this latter optimization also being based on the DM's utility). Consider the following definitions:

$$V = \{ u(x): \ x \in X \} \tag{B-7}$$

V' is the set of all non-dominated points in V

$$V(X') = \{ u(x): \ x \in X' \} \tag{B-8}$$

Now we will proceed to show that D_{MOOT} also equals D^*.

Lemma 1: Assume u is strictly isotone, i.e., $x < x' \rightarrow u(x) < u(x')$. Then $V' \subset V(X')$.

Proof: We wish to show that $v \in V'$ implies $v \in V(X')$. Assume $v \in V'$ but that $v \notin V(X')$. Since $v \in V' \subset V$, there exists, an $x \in X$ such that $v = u(x)$. Also $x' \in X' \rightarrow u(x') \in V(X')$ is equivalent to $u(x') \notin V(X') \rightarrow x \notin X'$; thus, $x \notin X'$. Now, $x \notin X'$ implies the existence of an $\bar{x} \in X'$ such that $x < \bar{x}$ and hence $u(x) < u(\bar{x})$, which contradicts the initial assumption that $v \in V'$. Thus, $v \in V'$ implies $v \in V(X')$. Q.E.D.

Lemma 2. Let $f: \ Z \rightarrow R$ be isotone on Z where Z is partially ordered ($\leq_Z$) and assume $Z' \subset Z$ is such that for any $z \in Z$ there exists a $z \in Z'$ such that $z \leq_Z z'$. Then

$$\sup_{z \in Z} f(z) = \sup_{z \in Z'} f(z) \tag{B-9}$$

Proof: Clearly, $Z' \subset Z$ implies

$$\sup_{z \in Z'} f(z) \leq \sup_{z \in Z} f(z) \ . \tag{B-10}$$

We wish to show the reverse inequality. Let (z_k) be a sequence in Z such that $f(z_k)$ converges up to $\sup_{z \in Z} f(z)$. For each k, choose $z'_k \in Z'$ such that $z_k \leq_Z z'_k$ and hence $f(z_k) = f(z'_k)$. The following equality - inequality string proves the result:

$$\sup_{z \in Z} f(z) \tag{B-11}$$

$$= \lim_k \operatorname{Inf} f(z_k) \tag{B-12}$$

$$= \lim_k \operatorname{Inf} f(z'_k) \tag{B-13}$$

$$= \sup_{z \in Z'} f(z) \; .$$

Q.E.D.

Theorem 1: Let u be strictly isotone and U be isotone on their respective domains. Then:

$$\sup_{x \in X} U(u(x)) = \sup_{x \in X'} U(u(x)) \; . \tag{B-14}$$

Proof: Clearly, since $X' \subset X$, $\sup_{x \in X'} U(u(x)) \leq \sup_{x \in X} U(u(x))$. We wish now to prove the reverse inequality. By definition,

$$\sup_{x \in X} U(u(x)) = \sup_{v \in V} U(v) \tag{B-15}$$

and

$$\sup_{x \in X'} U(u(x)) = \sup_{v \in V(X')} U(v) \tag{B-16}$$

It follows from Lemma 2 and the fact that for any $v \in V$ there exists a $v' \in V'$ such that $v \leq v'$ that implies

$$\sup_{v \in V} U(v) = \sup_{v \in V'} U(v) \tag{B-17}$$

Lemma 1 implies:

$$\sup_{v \in V'} U(v) \leq \sup_{v \in V(X')} U(v). \tag{B-18}$$

The above inequality and equalities then imply that

$$\sup_{x \in X} U(u(x))$$

$$= \sup_{v \in V} U(v) \tag{B-19}$$

$$= \sup_{v \in V'} U(v) \tag{B-20}$$

$$\leq \sup_{v \in V(X')} U(v) \tag{B-21}$$

$$= D_{MOOT} \tag{B-22}$$

$$= \sup_{x \in X'} U(u(x)) \tag{B-23}$$

$$= D^* \tag{B-24}$$

which proves the theorem. Q.E.D.

Therefore, the intended result is proven. It can be seen that the optimal policy is non-dominated.

When a ranking of alternatives is required as a solution to a decision situation, one is tempted to only consider the NDSS for the top rankings. Figure B-1 shows the two dimensional discrete case with three non-dominated solutions (a, b, and c) and one dominated solution d (b dominates d). The isopreference lines are formed through elicitation of the preference structure of the DM. As can be seen by comparison of the intersection of the alternatives and the isopreference lines, the dominated solution should actually be ranked ahead of many of non-dominated solutions. To investigate this condition, consider the following Lemmas and example.

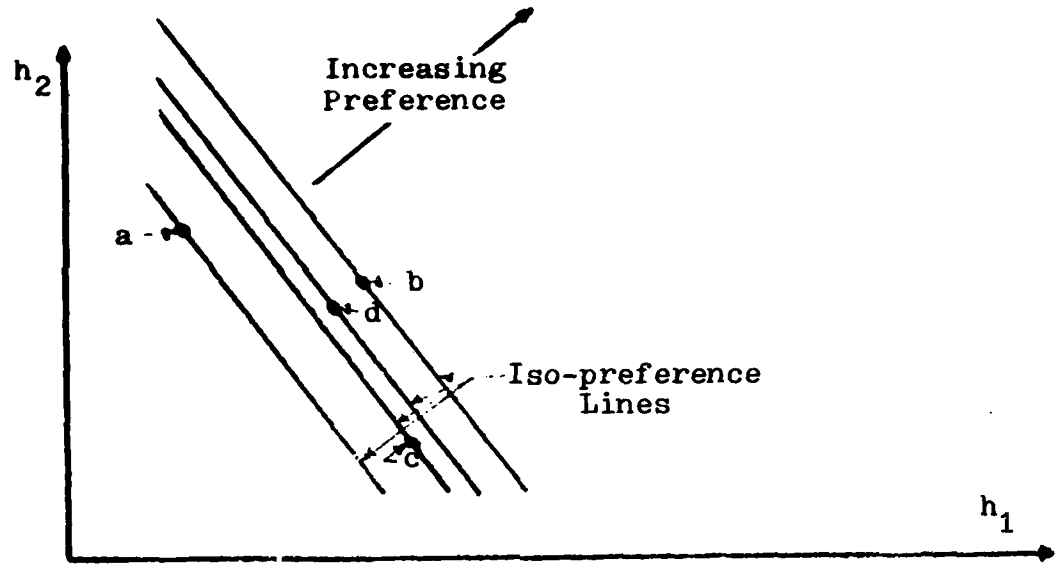
Lemma 3. Let x^* be the optimal alternative such that no other x in X dominates x^*. Let x_2 be the second best optimal alternative, where $x_2 \epsilon X$. Assume that an alternative policy $x_3 \epsilon X$ exists. If x_2 is dominated ($x_2 \not\epsilon X'$) by $x_3 \epsilon X$, then x_3 must equal x^* (i.e., x_2 can be dominated <u>only</u> by x^*).

Proof: From theorem 1, an optimal policy x^*, must be non-dominated. Hence $x^* \epsilon X'$, $v(x^*) \epsilon V'$, and from Lemma 1, $v(x^*) \epsilon V(X')$. For x_2 to be dominated, $x_3 \epsilon X$

$$v_i(x_3) > v_i(x_2) \text{ for some } v_i;\ i = 1,,,,n \tag{B-25}$$

$$v_j(x_3) \geq v_j(x_2) \forall j \neq i \tag{B-26}$$

If $x_3 \neq x^*$, then $x^* \epsilon X'$ by definition of the optimal policy. Since x_2 is the second best optimal, then the one alternative which dominates x_2 must be non-dominated. One is now faced with the following situation.



Attribute Space $h^T=(h_1,h_2)$

Figure B-1. Dominated And Non-Dominated Alternatives

Either x_2 is not dominated ($x_2 \epsilon X'$), or $x_3 = x^*$, or $x_3 \neq x^*$ which requires $x_3 \not\epsilon X$. If $x_3 \not\epsilon X$, this violates the original assumption. Thus, if the second best policy is dominated, it must be dominated by the optimal policy. Q.E.D.

Lemma 4. Let the second best alternative, x_2 be dominated by the optimal policy x^*. Let $x_2 \not\epsilon X'$. If x^* is eliminated, then x_2 now becomes non-dominated ($x_2 \epsilon X'$).

Proof: If $x_2 \not\epsilon X'$, then there exists one x (called x^*)

$$v_i(x^*) > v(x_2) \text{ for some } v_i;\ i = 1,,,,n \quad \text{(B-27)}$$

$$v_j(x^*) \geq v(x_2),\ \forall\ j \neq i \quad \text{(B-28)}$$

From Theorem 1 and Lemma 3, the optimal alternative is non-dominated. Therefore, eliminating x^* creates the condition of no $x \epsilon X$ which satisfies the two conditions above with respect to x_2, and so by definition, x_2 is now non-dominated. Q.E.D.

Theorem 1 shows the expected result that the optimal alternative is a member of the NDSS, but Lemmas 3 and 4 show that it is possible for the second best alternative to not be a member of the NDSS. As shown on Figure B-1, the preferences of the DM would rank all alternatives as b $\succ$ d $\succ$ c $\succ$ a. If only the non-dominated solutions were considered, the ranking of the alternatives becomes b $\succ$ c $\succ$ a. Transitivity would then call for us to compare the first (b) and third (c) or fourth (a) alternatives which is illogical since the obvious comparison which would make sense to the DM is between the first (b) and second (d) alternative. It is not difficult to imagine comparing the first and nth alternatives (when considering only the NDSS) for a system with a large number of alternatives. To prevent this condition, care must be taken to prevent eliminating an alternative which is not dominated by a majority of the alternatives. To be absolutely certain that a dominated alternative has not been inappropriately eliminated, the NDSS should be reformed without the optimal policy to see in any alternatives now enter NDSS before the new optimal is identified. Consider the following example.

Assume a company is interested in acquiring property. There are

several parcels of land available. The DM for the company has indicated that the attributes of importance concerning the property are size and quality. The Table below shows the alternatives (purchase area i) and associated attribute scores as supplied by the DM.

area	amount score (S_1)	quality score (S_2)
A	.8	.8
B	.7	.5
C	.5	.7
D	.2	.2
E	.9	.3

It can be seen that A dominates all alternatives except E,and D is dominated by all alternatives. It is patently clear that D should be eliminated, but what about B and C? By definition, alternatives A and E are the only members of the NDSS. Depending on the preference structure of the DM, either A or E would be selected as the optimal choice and the other would be assumed "second best." This last assumption may be erroneous as shown by the following calculations. For efficiency, we will choose a linear additive value function for the DM

$$V^i = kS_1^i + (1 - k)S_2^i \qquad \text{(B-29)}$$

where S_j^i is the jth attribute score for alternative i. Assuming values for the scaling constant k, the following results are obtained:

Considering solutions A, B, C, and E	Considering solutions A and E only	Scaling constant values
$A \succ C \succ B \succ E$	$A \succ E$	$0 \leq k < .5$
$A \succ B, C, E$	$A \succ E$	$k = .5$
$A \succ E \succ B \succ C$	$A \succ E$	$.5 < k < .83$
$A, E \succ B \succ C$	$A \sim E$	$k = .83$
$E \succ A \succ B \succ C$	$E \succ A$	$.83 < k \leq 1$

For the cast $0 \leq k \leq .5$, the non-dominated alternative E is not as good as either B or C, yet if B or C would have been eliminated for the single domination by A, for this set of scaling constant values, the first and

fourth ranked choices would have been compared. If for some reason alternative A would not have been implementable, then the DM would have been faced with selecteing the fourth alternative if the NDSS would not have been reformed. This example shows that one must be very careful when eliminating alternatives which are not dominated by a majority or all of the other alternates.

3. Consistency

The basis for part of the formulation in the general situation statement is the assumption that the DM is rational and consistent, and the optimum policy is a member of the possible set of decisions in the MAUT formulation. That is we assume that the DM's utility over the total attribute space $U(u_1(x),,,,u_n(x))$ is consistent with his or her utility over a reduced attribute space (defined by the NDSS) $U(v_1(x),,,, v_n(x))$. This assumption seems reasonable, and deviation from this consistency may cause variation in policy. It is noted that a slight variation in DM consistency will produce strategically equivalent policies from MOOT and MAUT which is the expected result of any techniques used to resolve a common decision situation (Keeney and Kirkwood, 1977).

4. Summary

The purpose of this appendix was to demonstrate the intuitively obvious result that MOOT and MAUT will identify strategically equivalent policy as optimal when both approaches are applied to a common decision situation, and to show that caution must be exercised in eliminating dominated alternatives from consideration in a selection process. It was assumed that a consistent DM is utilized in the application. A warning is issued that the above result may not hold if the elements of uncertainty, posterior equity considerations, and a group of DMs are present in the decision situation (Bodily, 1976).

Areas for continued research are identified by the need to further explore the problem of inconsistency of utility of DMs, (Tversky and Kahnman, 1977), and to formulate quantitative techniques to justify elimination of alternatives by domination in the case where an alternative is dominated by more than one other alternative. **(Appreciation is expressed to C.C. White and A.P. Sage for contributions to this effort.)**

APPENDIX C

ORGANIZATIONS CONTRIBUTING TO THE EWARD EFFORT

Air Force Systems Command
Aeronautical Systems Division
Wright-Patterson AFB, Ohio 45433

Defense Systems Management College
Policy, Analysis and Process Division
Fort Belvoir, Virginia 22060

Headquarters Air Force Systems Command
Electronic Warfare Acquisition Branch
Andrews AFB, Maryland 20331

Headquarters Department of the Air Force
Office of Deputy Chief of Staff for Research and Development
Washington, D. C. 20330

Headquarters U.S. Marine Corps
Code - AWN
Washington, D. C. 20380

Headquarters Strategic Air Command
Operations and Plans
Omaha, Nebraska 68113

Hughes Aircraft Corporation
Defense Systems
Canoga Park, California 91304

Naval-Air Systems Command
PMA/PME
Washington, D.C. 20361

Naval Avionics Center
21st Street and Arlington Avenue
Indianapolis, Indiana 46218

Naval Surface Weapons Center
Electromagnetic Systems Division
Dahlgren, Virginia 22448

Office of the Secretary of Defense
Defensive Systems Division
Washington, D.C. 20301

Raytheon Company
Electromagnetic Systems Division
Lexington, Massachusetts 02173

Several people who contributed to this effort requested that they not be identified personally because of the nature of this application. Consequently no names of individuals have been used here.

APPENDIX D

ELICITATION OF PREFERENCE STRUCTURES

Within the EWARD effort, elicitations were carried out with the decision makers (DM) and advisors in order to explore the relationships among the attributes with respect to preferential independence, utility independence, utility assessment and scaling constant magnitudes. The attributes and their ranges are listed in Table 4 of Chapter 5. The described elicitation process took place with a member of stakeholder Group 1, but similar processes also occurred with the members of Groups 2 and 3.

1. Preferential Independence: The following is an abbreviated set of questions and answers which were used in one case to establish preferential independence between aerodynamic performance ($\underline{X}_1$) and life cycle cost (X_2).

Analyst (A): Consider the attributes $\underline{X}_1$ and X_2 as shown on Figure D-1a. Assume all the other attributes are at their worst levels (Table 4, Chapter 5). Suppose you are at point A. Would you now prefer to move to point B or C?

Decision Maker (DM): Point B.

(A) What point along the x_1 = .75 line would be equally preferred to point B?

(DM) Point C'

(A) Now what point along the x_1 axis (x_2 at its best level) would be equally preferred to points B and C' (likewise find the equally preferred point along the x_2 axis)?

(DM) Point D (Point E)

This now forms a rough isopreference line L_1 as shown in Figure D-1b. This process is repeated starting at points A' and A" to form isopreference lines L_2 and L_3. Then the levels of all the other attributes except $\underline{X}_1$ and X_2 ($X_{\overline{12}}$) are changed first to intermediate levels, and then their highest levels, and the process repeated to form isopreference lines. If the isopreference lines on Figure 1-Db are the same regardless of the levels of $X_{\overline{12}}$), then the attributes X_1 and X_2 are

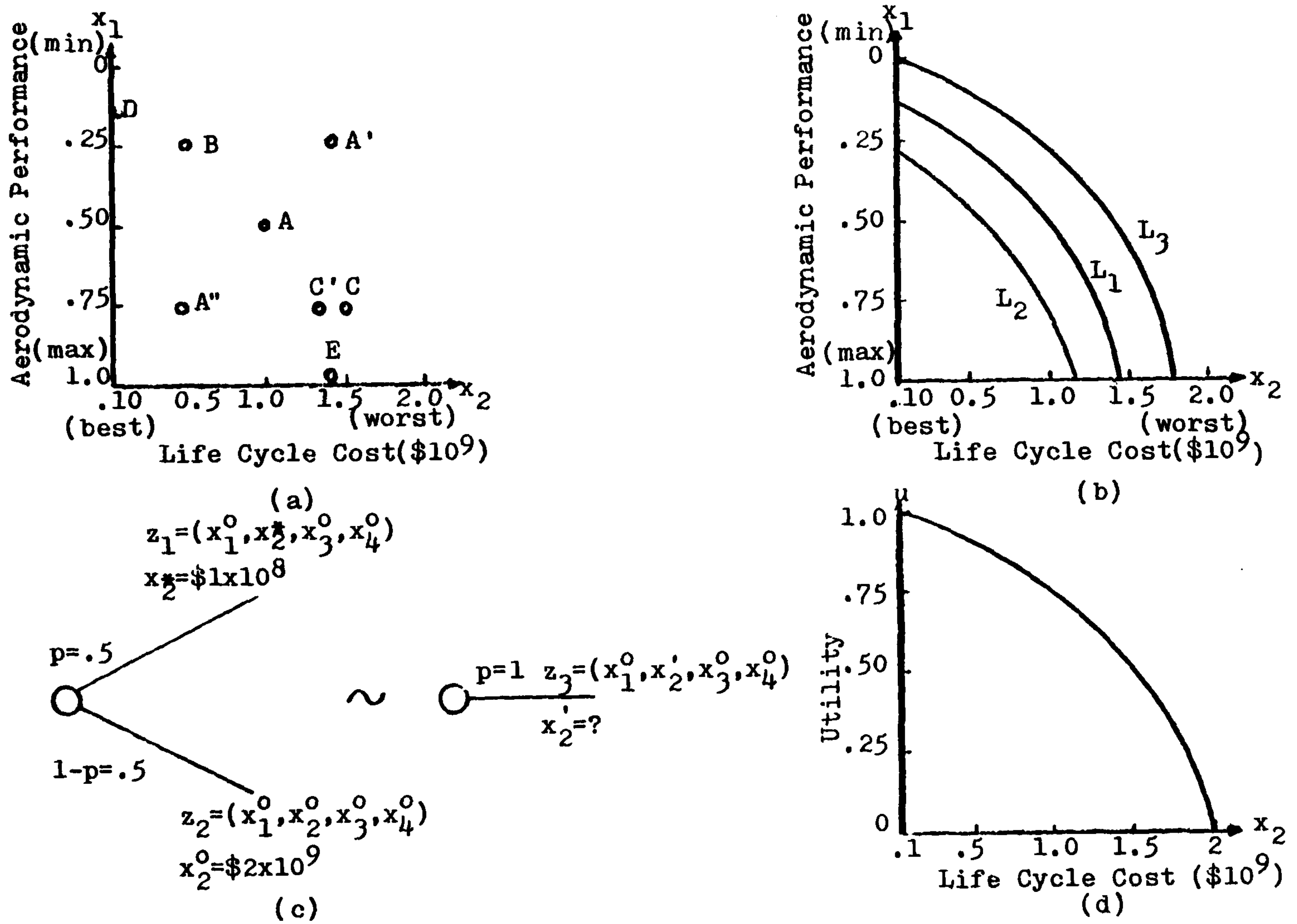


Figure D-1 Preference Structure

preferentially independent (PI) of the other attributes (X_1, X_2 PI$X_{\overline{12}}$ where $X_{\overline{12}} = X_3, X_4$ in this case). In this manner, mutual PI (MPI) was established by investigating PI for the various combinations of attributes for all higher level attributes $\underline{X}_1$, X_2, $\underline{X}_3$, X_4 and within the groups of lower level attributes (X_{1a}, X_{1b}, X_{1c}; and X_{3a}, X_{3b}, and X_{3c}).

2. Utility Independence and Utility Assessment: The condition of utility independence was established for the attribute of life cycle cost (X_2) and the utility of this attribute was assessed through a set of questions and a process which are summarized below.

(A) Consider the lottery shown on Figure D-1c. Set all the other attributes at their worst level except X_2, now would you prefer a lottery with a 50-50 chance of getting a system with minimum cost of $\$1 \times 10^8$ (assume the utility of this amount equals 1. $\Rightarrow u(\$1 \times 10^8) = 1.$) or system with maximum cost of $\$2 \times 10^9$ (assume $u(\$2 \times 10^9) = 0$), or for certain a system with cost of $\$1 \times 10^9$?

(DM) The certain return.

(A) What if the certain outcome was a system with a cost of $\$1.3 \times 10^9$, which would you prefer?

(DM) Still the certain outcome.

This continued until the certainty equivalence for the lottery $u(\$1.5 \times 10^9) = .5u(\$1 \times 10^8) + .5u(\$2 \times 10^9) = .5$ was established at a certain system with a cost of $\$1.5 \times 10^9$. This lottery process was repeated for different levels of the other attributes. The same choices resulted independent of the levels of the other lotteries, thereby establishing utility independences (UI) of X_2. Now that X_2 was established as UI, the utility of this attribute could be assessed independently of the other attributes. Now other lotteries were formed to define other certain equivalences and thereby define a utility curve. After iteration and corroboration by the group members, the utility curve for X_2 for stakeholder group 1 is shown on Figure D-1d.

From Theory 6.2 (Keeney and Raiffa, 1976), the conditions of MPI of the attributes and an attribute X_i being established as UI, establishes MUI for the groups of higher level attributes $\underline{X}_1$, X_2, $\underline{X}_3$, X_4 and lower level attributes X_{1a}, X_{1b}, X_{1c} and X_{3a}, X_{3b}, X_{3c}. This condition of MUI

was corroborated by investigation of UI of the various attributes in the manner described above. MUI allows the utility for each of the attributes to be assessed independently with a process as discussed previously for X_2.

3. Scaling constant evaluation: The weights for the various attributes were established through the abbreviated elicitation process now described. The multiplicative form of an aggregated utility function was established as a scalar choice function (as discussed in Section 4 of Chapter 5). For the aerodynamic performance attributes, this form is

$$1 + k_1u_1 = (1 + k_1k_{1a}u_{1a}(x_{1a}))(1 + k_1k_{1b}u_{1b}(x_{1b}))(1 + k_1k_{1c}u_{1c}(x_{1c}) \quad \text{(D-1)}$$

where k_1 is the aggregated scaling constant, u_1 is the aggregate utility function for aerodynamic performance, and k_{1i} is the individual attribute scaling constant , and u_{1i} is the utility function for the attribute levels of x_{1i} $(i = a,b,c)$.

First we established the ordering of k_{1a}, k_{1b}, and k_{1c} with respect to magnitude.

(A) Assume all lower level aerodynamic attributes are at their lowest level. Would you prefer to obtain a system which moves the system weight from 4500 kg down to 500 kg, or a system with volume of $1.5m^3$ instead of $4.1m^3$, or a system that demands only 45 KVA instead of 105 KVA?

(DM) I would opt for the system with the low volume (this indicates that $k_{1b} > k_{1a}, k_{1c}$).

(A) Now which choice would you prefer between weight and power systems?

(DM) I would next choose the system with low weight (this indicates $k_{1a} > k_{1c}$).

Therefore the order of magnitude for the scaling constants is $k_{1b} > k_{1a} > k_{1c}$.

(A) Now to establish numbers for these constants, assume you have two systems with the following characteristics: one system has the volume and power attributes at their worst level and the weight attribute at its best level $(x_{1a}^*, x_{1b}^\circ, x_{1c}^\circ)$ and the other system with the weight and power attributes at their worst level and the volume attribute at an unspecified level $(x_{1a}^\circ, x_{1b}', x_{1c}^\circ)$. What should x_{1b}' equal for the two

systems to be equally preferred?

(DM) $x_{1b} = 2.4m^3$

Substituting these values into eq. (D-1) yields

$$k_2 u_2(2.4) = k_1 \tag{D-2}$$

where u_2 can be obtained from the utility curve for cost. The question was repeated for the power attribute:

(A) What level of x_{1b} would make systems with characteristics of

$$x_{1a}^{\circ},\ x_{1b}^{\circ},\ x_{1c}^{*} \text{ equally preferred to } x_{1a}^{\circ},\ x_{1b}^{''},\ x_{1c}^{\circ}\ .$$

(DM) $$x_{1b}^{''} = 3.5$$

Substituting these values into eq. (D-1) yields

$$k_2 u_2(3.5) = k_3 \tag{D-3}$$

(A) Now for what probability, p*, will a lottery between systems with attributes of x_1^*, x_2^*, x_3^* with probability p* and x_1°, x_2°, x_3° with probability 1-p* be equally preferred to a system with attributes x_1°, x_2^{*}, x_3° for certain?

(DM) p* = .59

Substituting into eq. (D-1) yields

$$p^* = k_{1b}$$

Now for consistency, $u_1(x_{1a}^{\circ}, x_{1b}^{\circ}, x_{1c}^{\circ}) = 0$ and

$$u_1(x_{1a}^{*}, x_{1b}^{*}, x_{1c}^{*}) = 1 \tag{D-4}$$

which when substituted into eq. (D-1) yields

$$1 + k_1 = (1 + k_1 k_{1a})(1 + k_1 l_{1b})(1 + k_1 k_{1c}) \tag{D-5}$$

Now we have four variables (k_1, k_{1a}, k_{1b}, k_{1c}) and four equations (D-2, D-3, D-4, D-5) and hence all the variables can be evaluated to yield $k_{1a} = .48$, $k_{1b} = .59$, $k_{1c} = .24$, $k_1 = -.6$.

These exercises were repeated for all groups with the results shown on Tables 8 and 9 of Chapter 5.

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